

## EFFECTIVE DECISION MAKING FOR PRESENCE OF SCOLIOSIS<sup>¶</sup>

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### ABSTRACT

Scoliosis, a body-disfiguring disease, is associated with lateral curvatures and rotations of a person's spine. It is, generally, detectable around the age of 8 years. A two-minute-stripped-orthopedic examination of students, in the age group seven- to ten-years, may alert the health-care provider to early-warning signals, which are expressed as a mathematical index, named as 'Cumulative-Scoliosis-Risk Weightage (CSRW)'. CSRW is based on family history, age, statuses of being tall and/or wasted, forward-bending tests, non-alignment of plumb-line, shoulder drooping, uneven scapulae, shape of midline of back, unequal body triangles, uneven spinal dimples and positive moiré. A high CSRW calls for further examination before sending the child for X rays. Effective methods are needed to eliminate need for unnecessary X rays, which damage bone marrow of children. A mathematical model is proposed and tested on seven- and eight-year old students of a local school. Four tests were conducted, visual (standing), visual (sitting), forward bending (standing) and forward bending (sitting) — postural problem suspected through positive visual examinations (standing and sitting), indicated through positive mid-stretching test; leg-length in-equality suspected through positive visual and forward-bending tests (both standing), indicated through uneven spinal dimples; hip weakness suspected through positive visual and forward-bending tests (both sitting), indicated through positive Tredelenburg sign; spinal rotation suspected through positive forward-bending tests (standing and sitting), indicated through positive moiré. This paper reports effectiveness of CSRW in predicting lateral curvatures and spinal rotations.

**Keywords:** Modeling of spinal column, spinal-deformity screening, cumulative-scoliosis-risk weightage

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### LIST OF ABBREVIATIONS

*mm*: millimeter(s) • *cm*: centimeter(s) • *kg*: kilogram(s)

**AP:** Anteroposterior  
**CSRW:** Cumulative-Scoliosis-Risk Weightage  
**NGDS:** National Growth and Developmental Standards for the Pakistani Children  
**SF:** The Syed Firdous Growth-and-Imaging Laboratory, University of Karachi  
**SGPP:** Sibling Growth Pilot Project — a subproject of the NGDS Pilot Project

### INTRODUCTION

From the ancient civilization man was interested in maintaining an upright posture. Military parades and exercises, also, serve the same purpose. The original meaning of 'orthopedics' is straight (ortho) child (pedics). Hence, surveillance of scoliosis in primary-school children is of utmost importance (Persson-Bunke *et al.*, 2012; Saikia *et al.*, 2002). Scoliosis is defined as lateral curvatures and rotations of the spinal column and is commonly checked through visual examination of back in the attention position and Adam's forward-bending test (Anderson, 2007; Sengupta and Webb, 2010).

The above-mentioned tests generate a large number of false positives in young children under the age of 11 years. These children are, then, subjected to X rays to either confirm or rule out scoliosis. Unnecessary X-ray exposure to children, who have a delicate bone marrow, is of concern to clinicians and they have been looking for alternate methods to limit the number of children, who are sent for X rays. Moiré fringe topography is one such technique, which highlights minimal left-right asymmetries on the back of child using ordinary light (non-ionizing radiation). In fact, AP-X ray of the spinal column and moiré fringe topography of the entire back provide different sets of information about the spinal column — the first one shows lateral curvatures and the second one rotations. Hence, it is recommended that both of these techniques should be used in conjunction to obtain a complete picture.

The world of orthopedic surgeons is going beyond study of localized curvatures to modeling of the entire spinal column, all the way to vertebral and sub-vertebral levels. This paper lists some attempts in this direction as well as mathematical modeling for scoliosis-risk indicators and a decision matrix to select cases for follow up using moiré fringe topography, radiography and other imaging techniques.

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## MODELING OF THE SPINAL COLUMN

Anthromathematics of the human spinal column is going to be one of the most active areas of research in this century, which should involve modeling of the spinal column (Kamal *et al.*, 2013c).

### 2-D Modeling

2-D-spinal-column models should be able to generate frontal view from projections of spine obtained from AP-X rays or back moiré topographs in the attention position (Oxborrow, 2000). An attempt was made to determine Cobb angles from moiré topographs of back (Kamal, 1982a; El-Sayyad and Kamal, 1981).

### Need for 3-D Modeling

Spinal column is a structure, which exists in three dimensions. AP-X rays of the spinal column show the entire spinal column from the external auditory meatus to hip joint (patient in the attention position). These X-ray pictures exhibit spinal projection only in the frontal plane. Hence, they are not capable of showing kyphosis or lordosis. 3-D-spinal-column models should have the ability to synthesize full view from projections of spine in the frontal and the sagittal planes, generated from AP- and lateral-X-ray pictures or moiré topographs of back in the attention position (Kamal 1982b). Recently, Bella *et al.* (2014) have tried to define shape of spine using moiré method.

### 3-D-Static Modeling

3-D static modeling was started in simultaneously in 1982 in Germany (Hierholzer and Lüxmann, 1982) and in USA (Kamal, 1982b; 1983 a; b). Natural curvatures of the spinal column, as seen in the sagittal-plane projection, were incorporated in a later work (Kamal, 1987). A comprehensive model was published in 1996, which is, briefly, described here (Kamal, 1996c). From X-ray or moiré measurements, a parabolic curve was generated, relating  $x$ ,  $y$  and  $z$ , where  $x = x(\xi)$ ,  $y = y(\xi)$ ,  $z = z(\xi)$ , best fitted to discrete measurements performed at different locations represented by the parameters,  $\xi_i$ ;  $i = 1, \dots, 33$ ; corresponding to 33 vertebrae of backbone, consisting of cervical, thoracic, lumbar and sacral regions. The parabolic curve was represented by

$$(1) \quad x = f(y, z) = \frac{1}{2}ay^2 + byz + \frac{1}{2}cz^2$$

The cross term ( $yz$ ) vanished if the coördinate system was rotated through an angle  $\alpha$  given by

$$(2) \quad \alpha = \frac{1}{2} \tan^{-1} \frac{2b}{c-a}$$

Coefficients of squares of  $y$  and  $z$ , then, gave half of curvatures. Degree of correction of spinal deformity was defined in terms of these curvatures, taking into account normal curvatures of the human spinal column. Trunk deformity was classified as ‘severe’, ‘intermediate’ or ‘mild’ depending on the value of degree of correction of spinal deformity in the ranges 0-33.33, 33.34-66.66, 66.67-100, respectively. This 3-D-static model was found to be useful in the study of posture of children.

### 3-D-Dynamic Modeling

The dynamic model was a generalization of 3-D-static model, to study movement of the human spinal column during a gait cycle (Kamal, 1996a; Yosufzai *et al.*, 1995). The human spinal column in three dimensions was generated from moiré topographs of back in the attention position as well as during each of the four steps of the gait cycle (Kamal, 1996d). Spinal column in the attention position was, then, linked to first step through edge-based algorithm (Kamal, 1997b). Similarly, position in the second step was linked to the first step through edge-based algorithm and so on.

### 3-D-Crystal-Structure-Based Modeling

The human spinal column was considered as a crystal structure — a collection of vertebrae in the cervical, the thoracic, the lumbar and the sacral regions, which are located at specific distances from each other. The center-of-mass of each vertebra was expressed in terms of positional coördinates ( $x$ ,  $y$ ,  $z$ ) in the body-coördinate system. It could be considered as ‘form factor’, being used in crystallography. Study of vertebral-surface structure using moiré fringe topography, which provides height map of surface of child’s back (Kamal *et al.*, 2013b), rasterstereography, which gives local curvatures at various points of child’s back (Hackenberg *et al.*, 2003a; b; 2006; Kamal *et al.*, 2013a) or the enhanced version, dotted-rasterstereography, which generates better information of these curvatures (Wasim *et al.*, 2013) as well as combined moiré, raster and backscatter-X ray (Kamal, 2013; Kamal *et al.*, 2014b), which draws height and curvature maps on spinal-column surface. If rotational (in terms of Euler angles) and inter-

vertebral-spacing information is added, the analysis may be visualized as ‘structure factor’, used by solid-state physicists to study crystal structure (Kamal *et al.*, 2012).

### CUMULATIVE-SCOLIOSIS-RISK WEIGHTAGE (CSRW)

There is a dire need to study factors associated with scoliosis in school children (Baroni *et al.*, 2015). A mathematical index ‘Cumulative-Scoliosis-Risk Weightage’ (CSRW) was devised by our group, which assigned a weight to each early-warning signal, *e. g.*, family history (scoliosis in parents or one of the siblings increases the risk), age group, tallness, wasting, positive forward-bending tests, plumb-line non-alignment, positive indicators in visual examination of back (drooping shoulders, uneven scapulae, curved shape of midline of back, unequal body triangles, uneven spinal dimples) and positive moiré (front and back), with the weightage increasing if the condition exists for more than one checkup. Table 1 lists all these factors and their respective weightages for a single or multiple checkups (Kamal *et al.*, 2013d).

Differential-spinal-function testing (see the following section) should be performed to rule out scoliosis if CSRW is equal to or more than 5.0 after 1<sup>st</sup> checkup, 7.0 after 2<sup>nd</sup> checkup and 9.0 after 3<sup>rd</sup> checkup.

### DIFFERENTIAL-SPINAL-FUNCTION TESTING

A mandatory two-minute-stripped-scoliosis screening, which includes moiré examination of back, for school-going children in the age group 7-10 years and a follow-up of at-risk cases may prevent suffering of a lifetime (Akram and Kamal, 1991; Horn, 2012; Luk *et al.*, 2010; Kamal and Lindseth, 1980).

Table 1. Formulae for Assigning Cumulative-Scoliosis-Risk Weightage (CSRW)<sup>§</sup>

<i>Scoliosis-Risk Weightage</i>	<i>A</i> <sup>‡</sup>	<i>B</i> <sup>€</sup>	<i>C</i> <sup>‡</sup>
01. Family history	2.0	2.0	2.0
02. Age [3, 6.5) <sup>‡</sup> years	0.5	0.5	0.5
03. Age [6.5, 7.5) <sup>‡</sup> years	1.0	1.0	1.0
04. Age [7.5, 8.5) <sup>‡</sup> years	1.5	1.5	1.5
05. Age [8.5, 11) <sup>‡</sup> years	2.0	2.0	2.0
06. Tall (above 50 <sup>P</sup> ) <sup>#</sup>	1.0	1.5	2.0
07. Tall (above 75 <sup>P</sup> ) <sup>#</sup>	1.5	2.0	2.5
08. Tall (above 97 <sup>P</sup> ) <sup>#</sup>	2.0	2.5	3.0
09. Wasted (more than 10%) <sup>#</sup>	1.0	1.5	2.0
10. Wasted (more than 20%) <sup>#</sup>	1.5	2.0	2.5
11. Wasted (more than 30%) <sup>#</sup>	2.0	2.5	3.0
12. FBT <sub>F</sub> (lumbar asymmetry)	1.0/1.5 <sup>€</sup>	1.5/2.0 <sup>€</sup>	2.0/2.5 <sup>€</sup>
13. FBT <sub>B</sub> (thoracic asymmetry)	1.0/1.5 <sup>€</sup>	1.5/2.0 <sup>€</sup>	2.0/2.5 <sup>€</sup>
14. Plumb-line non-alignment	1.0	1.5	2.0
15. Shoulder drooping	0.5	1.0	1.5
16. Uneven scapulae	0.5	1.0	1.5
17. Midline of back C-shaped	0.5/1.0 <sup>¥</sup>	1.0/1.5 <sup>¥</sup>	1.5/2.0 <sup>¥</sup>
18. Midline of back S-shaped	1.0/1.5 <sup>¥</sup>	1.5/2.0 <sup>¥</sup>	2.0/2.5 <sup>¥</sup>
19. Unequal body triangles	0.5	1.0	1.5
20. Uneven spinal dimples	0.5	1.0	1.5
21. Positive moiré (back)	1.0	1.5	2.0
22. Positive moiré (front)	0.5	1.0	1.5

<sup>§</sup>The student should be subjected to differential-spinal-function testing if CSRW is equal to or more than 5.5 after the first checkup, 6.5 after the second checkup and 7.5 after the third checkup

<sup>‡</sup>1<sup>st</sup> checkup or 2<sup>nd</sup> checkup or 3<sup>rd</sup> checkup

<sup>€</sup>(1<sup>st</sup> + 2<sup>nd</sup>) checkup or (2<sup>nd</sup> + 3<sup>rd</sup>) checkup or (1<sup>st</sup> + 3<sup>rd</sup>) checkup

<sup>‡</sup>(1<sup>st</sup> + 2<sup>nd</sup> + 3<sup>rd</sup>) checkup

<sup>‡</sup>[x, y) means x (3 years in the first entry) is included, but y (6.5 years) is not. Hence, a 6.5-year old student is rated according to criterion 03.

<sup>#</sup>The superscript P denotes percentile

<sup>€</sup>Second value is applicable, if the front and the back asymmetries are on the opposite side

<sup>¥</sup>Second value is applicable, if the deformity is not corrected upon asking the child to assume mild-stretching posture

### **Mathematical Concepts Involved in Scoliosis Screening**

Scoliosis screening would generate interest in the students to be screened (end users of this exercise), if they are taught in a classroom lesson (prior to being subjected to screening) to visualize anatomy of the spinal column as a geometrical figure, offering them opportunity to measure Cobb angles as part of their mathematics classes (Kurz *et al.*, 2015).

Kamal *et al.* (1998) discussed physics of scoliosis screening. Here, we list some of the mathematical concepts involved in scoliosis screening (Kamal, 2011):

**Symmetry:** Symmetry about sagittal plane (left-right) is the main concept behind screening for scoliosis — in the context of visual exam, the screener observed scapulae, body triangles, spinal dimples, shoulders/neck line, nipples and knee joints. If transverse axis were taken as the  $x$  axis, anteroposterior axis as the  $y$  axis and longitudinal axis as the  $z$  axis, sagittal plane could be identified as the  $yz$  plane. Mathematically, symmetry about the  $yz$  plane (described by the equation,  $x = 0$ ) was expressed as the condition — for every point  $(x, y, z)$ , which was on body surface, there existed a mirror point  $(-x, y, z)$  on the same surface.

**Inverse Problem:** The screener tried to determine properties of source (condition of spinal column) from the properties of field generated, *e. g.*, thermogram indicating crooked spine or determination of shape of organ from back-surface analysis using moiré fringe topography or rasterstereography

**Precedence Graph:** Precedence Graph illustrates the procedures, which must ‘precede’ the others, *e. g.*, heart was checked before Adam’s forward bending test. The later was omitted for a patient, who underwent multiple cardiac surgeries.

**Influence Graph:** Influence Graph shows various procedures ‘influenced’ by others. Visual examination of spine (in the attention position) did not give proper results, when the student was tired after vigorous activity, *e. g.*, right after recess or physical-education class.

### **Decision Matrix for Presence of Scoliosis**

Figure 1 illustrates decision matrix to detect possible existence of lateral curvatures and rotation of the spinal column. The decision was made in two levels. In the first level 2 tests were conducted and the results compared to suspect a possible condition. Then a 3<sup>rd</sup> test was administered to indicate that condition. 4 tests were selected for this purpose: visual (standing), visual (sitting), forward bending (standing) and forward bending (sitting) — postural problem suspected through positive visual examinations (standing and sitting), indicated through positive mild-stretching test (if the deformity was not corrected after mild stretching, it was indicative of lateral curvatures); leg-length in-equality suspected though positive visual and forward-bending tests (both standing), indicated through uneven spinal dimples; hip weakness suspected though positive visual and forward-bending tests (both sitting), indicated through positive Tredelenburg sign; spinal rotation suspected through positive forward-bending tests (standing and sitting), indicated through either positive moiré or positive forward-bending tests (back and front views) on opposite sides (Kamal *et al.*, 2014a).

## **MATERIAL AND METHODS**

### **Data Collection**

In 1998, the NGDS (National Growth and Developmental Standards for the Pakistani Children) Pilot Project (<http://ngds.uok.edu.pk>) was launched after ‘Institutional Review Process’, which incorporated existing ethical and human-right standards applicable in Pakistan employing ‘opt-in policy’. Participation was possible after filled in and signed ‘Informed Consent Form’ ([http://www.ngds-ku.org/ngds\\_folder/Protocols/NGDS\\_form.pdf](http://www.ngds-ku.org/ngds_folder/Protocols/NGDS_form.pdf)) was received. Those students, who required further examinations, were called in SF-Growth-and-Imaging Laboratory, along with their parents and siblings. ‘SGPP Participation Form’ ([http://www.ngds-ku.org/SGPP/SGPP\\_form.pdf](http://www.ngds-ku.org/SGPP/SGPP_form.pdf)) was filled out for detailed checkup — SGPP (Sibling Growth Pilot Project) is a subproject of the NGDS Pilot Project ([http://www.ngds-ku.org/ngds\\_URL/subprojects.htm#SGPP](http://www.ngds-ku.org/ngds_URL/subprojects.htm#SGPP)), in which Growth-and-Obesity Profiles of all siblings as well as Obesity Profiles of parents are generated (Kamal *et al.*, 2011). This paper reports analysis of spinal examinations performed on the students of a civilian school located in Karachi for the checkups performed during 2011-2013. Students studying in KG (4- to 5-year old) were enrolled in the study. They were followed up till they left class 2 (7- to 8-year old). Data of 68 boys and 65 girls are presented here. Posture examinations and brief scoliosis screening was performed in KG and Class 1 to find out cases of early onset of scoliosis (Fletcher and Brace, 2012; Tis *et al.*, 2012) and a detailed scrutiny of the spinal column performed in class 2 (Kamal *et al.*, 2014a).

Students were required to undress totally except briefs or panties. Spinal examinations were performed in standing position — visual and forward bending. Visual examination of back in the attention position consisted of

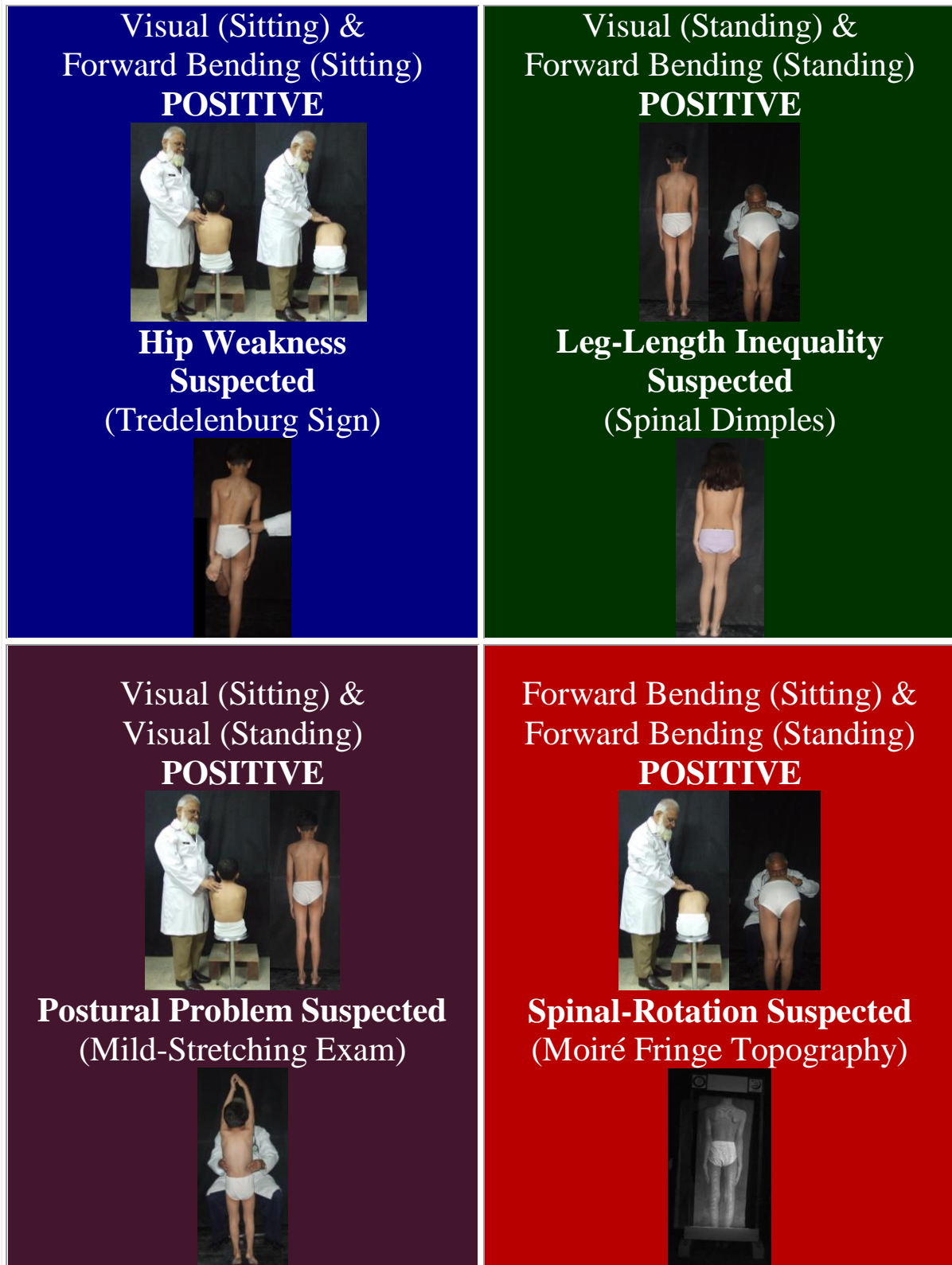


Fig. 1. Decision matrix to detect possible presence of spinal rotation, which may contribute to scoliosis, based on 4 tests, visual (standing), visual (sitting), forward bending (standing) and forward bending (sitting)

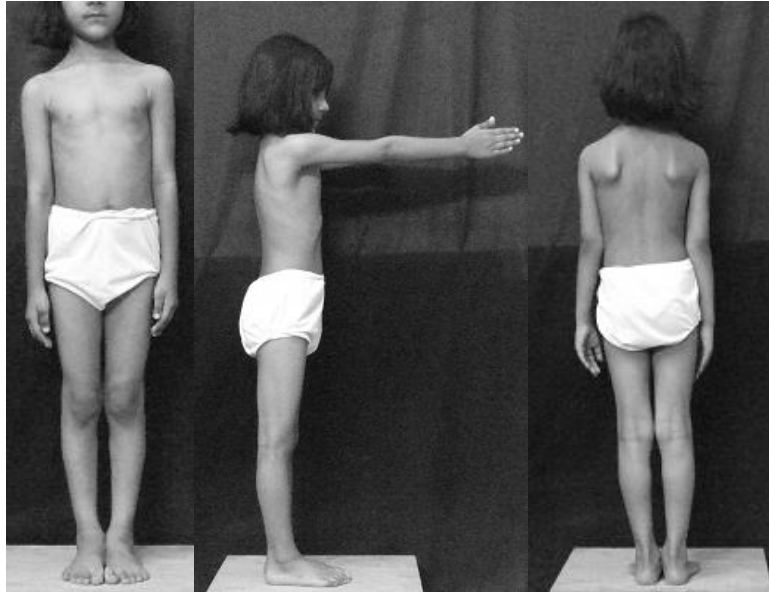


Fig. 2. Visual examination from front — drooping shoulder, nipples not level, side and back — drooping shoulder, uneven scapulae

noting presence of drooping shoulders, level of scapulae, symmetry of body triangles, level of spinal dimples and shape of midline of back — straight, C (most probably due to postural problem) or S (pathological — may be indicative of scoliosis). In addition, visual examination of child facing the examiner was conducted to confirm shoulder drooping, unequal body triangles and shape of sternum (if observed during back examination) as well as level of nipples and knee joints (Figure 2). Body alignment was checked by placing a plumb line along midline of back. Body as considered to be aligned from back if the plumb line passed through midpoint of spinal dimples. From the front, plumb line was aligned with sternum. Body as considered to be aligned from front if the plumb line passed through naval (Figure 3). Forward-bending test (standing position) was performed both with the student facing the examiner and with back towards the examiner. Student was asked to touch (or try to touch) toes with palms together and without flexing knees. The former brought into light curves in the lumbar and the sacral regions,



Fig. 3. Photograph on left illustrates body-alignment check of back using plumb line, body aligned, however, C curve is noticeable in the spinal outline and body triangles are unequal, photograph on right shows body-alignment check from front, nipples not level, body-triangles unequal and body not aligned



Fig. 4. Forward-bending test from front (lumbar region highlighted) — left side elevated, side and back (thoracic region highlighted) — right side elevated; S curve suspected

whereas the later highlighted curves in the cervical and the thoracic regions. The asymmetry observed in forward-bending test (student facing the examiner or back towards the examiner) was confirmed by observing the student in forward-bending position observed from the side opposite to elevated back, *i. e.*, a student showing left side of back elevated was observed from the right side and vice versa. The side observation highlighted elevated portion (Figure 4 — figure on extreme left shows forward-bending test, with student facing the examiner, in which left side is elevated, whereas figure on extreme right shows forward-bending test, with student's back towards the examiner, in which right side is elevated; strong indication of S curve and rotation of the spinal column). Students were asked to lift each foot for a count of 3 to check for hip weaknesses — the pelvis tilted downward on the side of unaffected hip, when weight was borne on weak hip abductors (positive Trendelenburg sign). Uneven spinal dimples indicated leg-length inequality.

In the sitting position visual and forward-bending examinations were performed by asking the child to sit on a stool with back straight. To make sure that the thighs were positioned perpendicular to back and feet were not hanging, wooden planks were placed under the feet to stabilize sitting posture. Body triangles could not be in the sitting-position-visual examination.

Moiré examination (Figure 5) was performed by asking the child to stand behind a shadow-type moiré frame of opening  $57 \times 133 \text{ cm}$  (grating dimensions  $52 \times 108 \text{ cm}$ ) constructed from fishing line of black nylon thread of



Fig. 5. Use of moiré fringe topography to look for symmetric curves about the sagittal plane of child's back and front for possible detection of scoliosis

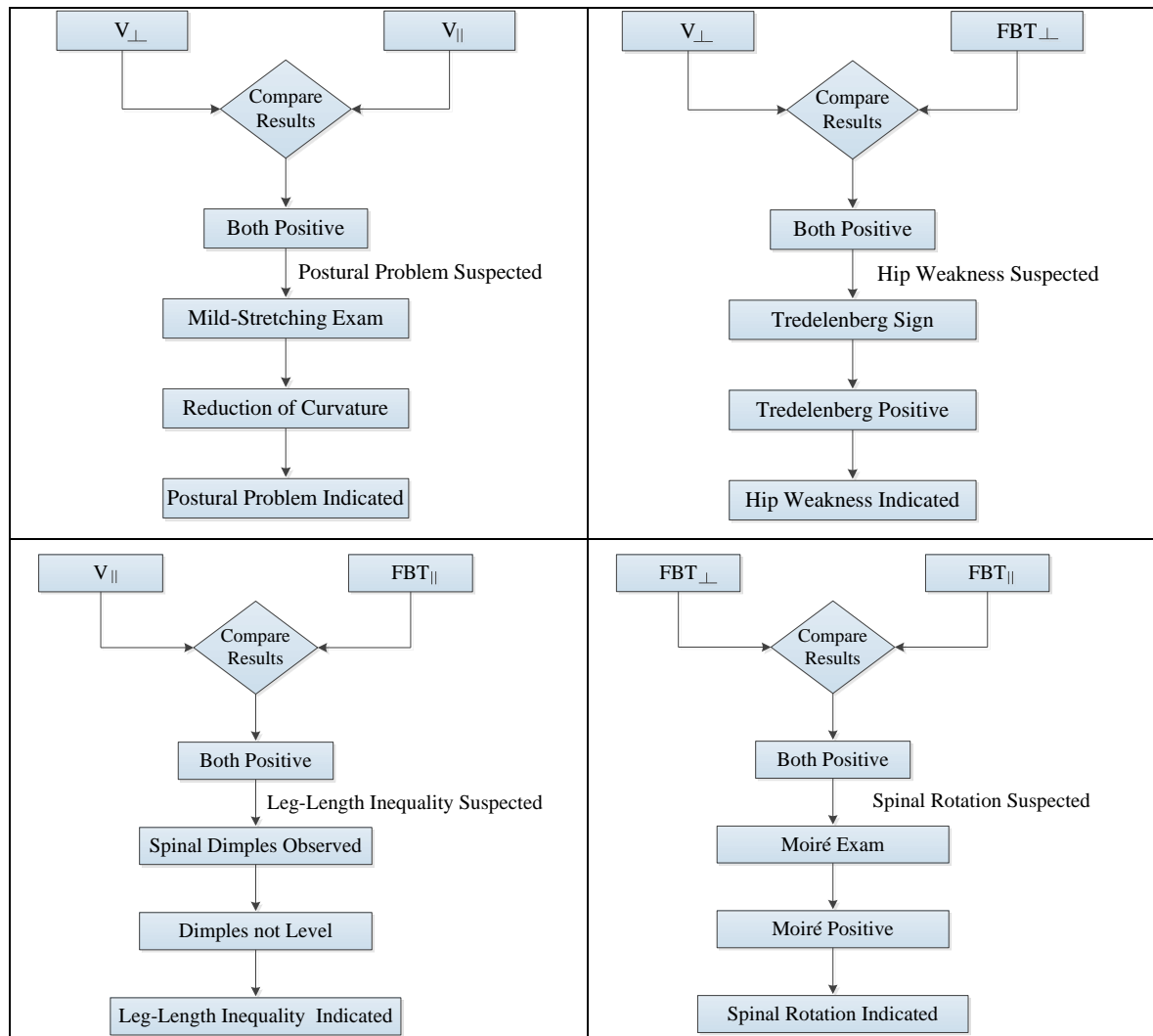


Fig. 6. Flowchart for decision matrix, which may be used to plan for efficient detection and effective treatment of scoliosis

diameter 0.85 mm wound along the longer side with spacing maintained through a spring of pitch 0.75 mm. Distance between the camera and the moiré grating was kept as 170 cm and that from the light source and the camera as 70 cm (Akram, 1989). Overhead projector was used as light source. A transparency was placed next to moiré grating to focus the light beam. The transparency was, later removed and the child was instructed to stand in the attention position touching the moiré grating.

Heights and masses of students were measured by reproducible anthropometrists to accuracies of 0.01 cm and 0.01 kg, in the morning hours, as per protocols described elsewhere (Kamal *et al.*, 2011; 2015). Equipments were calibrated using a standard 100-cm ruler and a standard 2-kg mass. Zero errors were determined before starting each session and subtracted from the measured values.

### Data Processing

Figure 6 gives a flow chart of the decision-making process (Kamal *et al.*, 2014a). We would like to study correlation of those cases in which lateral curvatures, spinal rotations or both were indicated with CSRW. Let us denote

$A$  : set of students, in whom lateral curvatures of spinal column were indicated

$B$  : set of students, in whom rotations of spinal column were indicated

$A \cap B$  : set of students, in whom both lateral curvatures and rotations were indicated

Table 2. Mean Cumulative-Scoliosis-Risk Weightage (CSRW) for students suspected of scoliosis

Differential-Spinal-Function Testing Result	Number of Boys	Number of Girls	Total	Mean CSRW <sup>∇</sup>
Lateral curvatures indicated, no rotations	1	2	3	11.92
Rotations indicated, no lateral curvatures	27	50	77	8.98
Both lateral curvatures and rotations	0	1	1	7.50

<sup>∇</sup>Mean of individual CSRW scores of each student

$A - B$  : set of students, in whom lateral curvatures were indicated, but rotations were not

$B - A$  : set of students, in whom rotations were indicated, but lateral curvature were not

From the above description, it could be easily concluded

$$(3) \quad A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$$

Arithmetic mean of CSRW was computed corresponding to elements of each set.

## RESULTS

Figure 7 shows the results of differential-spinal-function testing. Table 2 gives the number of students, who indicated lateral curvatures or rotations and the corresponding mean Cumulative-Scoliosis-Risk Weightage (CSRW). This analysis indicates that a higher CSRW increases the risk of student acquiring scoliosis — girls are approximately at a double risk of acquiring scoliosis as compared to boys.

## DISCUSSION AND CONCLUSION

The authors recommend posture examinations for boys and girls in the age group 4-7 years (Kamal and El-Sayyad, 1981) and scoliosis-screening examinations for children in the age group 7-10 years (Kamal *et al.*, 2013b) employing visual inspection and moiré fringe topography. Child should be totally stripped except short underpants and barefoot for these exams (Kamal *et al.*, 2015). At-risk cases, determined through high CSRW (5.5 after the first

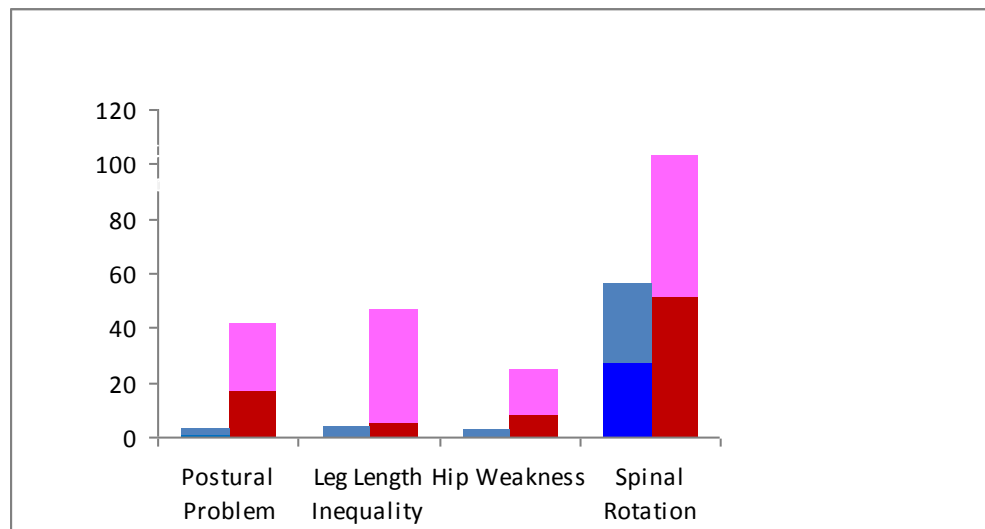


Fig. 7. Data analysis based on decision matrix, suspected conditions illustrated in indigo (boys) and pink (girls), indicated conditions in blue (boys) and red (girls) — postural problem suspected through positive visual examinations (standing and sitting), indicated through positive mid-stretching test; leg-length inequality suspected through positive visual and forward-bending tests (both standing), indicated through uneven spinal dimples; hip weakness suspected through positive visual and forward-bending tests (both sitting), indicated through positive Trendelenburg sign; spinal rotation suspected through positive forward-bending tests (standing and sitting), indicated through either positive moiré or positive forward-bending tests (back and front views) on opposite sides

checkup, 6.5 after the second checkup and 7.5 after the third checkup), should be followed up till the end of growth period (An *et al.*, 2015; Schulte *et al.*, 2008).

Future work should be focused on formulating a coördinate system, which should reduce the degrees-of-freedom of spinal column from a total of  $231 = (33)(3 + 3 + 1)$  to, possibly, one by employing techniques similar to those used in reducing degrees-of-freedom of two-body problem of planetary motion from 12 to one (Kamal, 1997b). The product  $(33)(3 + 3 + 1)$  comes from multiplying the number of vertebrae in the spinal column (33) with the sum of positional degrees-of-freedom (3), rotational degrees-of-freedom (3) + inter-vertebral-spacing degree-of-freedom (1). The problem of human spinal column, one of the leading problems of ‘orthopedics’ (considered as a branch of medical science dealing with the study of bones, joints and skeletal deformations), therefore, is described by the branch of mathematics called ‘algebraic topology’. One must realize that ‘orthopedics’ is derived from ‘anatomy’ (study of structures of human body), whereas ‘algebra’ is the understanding of mathematical structures and ‘topology’, studies invariance under deformations. ‘Anthrotopology’ may be the new branch of mathematics, which could deal with the mathematical framework related to spinal deformities (Kamal *et al.*, 2012).

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