INT. J. BIOL. BIOTECH. **13 (4): 649-669, 2016**

GROWTH-AND-OBESITY VECTOR-ROADMAPS OF THE PAKISTANI CHILDREN[¶]

Syed Arif Kamal[*], Ashfaq Ali Naz and Shakeel Ahmed Ansari[$]

*Department of Physics, University of Karachi, Karachi 75270, Pakistan; *e-mail***:** sakamal@uok.edu.pk

ABSTRACT

Obesity is becoming global problem among youngsters, with rise in incidence of childhood obesity throughout the world. This paper fine-tunes earlier mathematical solutions of childhood obesity, proposed during 2013-2016, by fitting a parabolic curve to desired percentile trajectories originating at the age of the most-recent checkup and terminating at 10 *years* (lower limit of the peripubertal phase). Both height and mass percentiles are made to approach the reference percentile (maximum of height percentile at the most-recent checkup, the mid-parental percentile, computed from the target height, and the army-cutoff percentile, 2.72 for the Pakistani boys and 19.36 for the Pakistani girls), as the child nears 10^{th} birthday. As compared to 'Growth-and-Obesity Scalar-Roadmap', proposed last year, which tries to cover up height and mass deficiencies within half-a-year, 'Growth-and-Obesity Vector-Roadmap', proposed in this paper, sets up softer targets for height and mass management during a time span of 6 *months*. These targets are expected to avoid unwanted stress on the body of a youngster and are in harmony with the fact that height gain represents tissue synthesis and must be managed through a process, which is quasi-static. 'Growth-and-Obesity Vector-Roadmap' is constructed from a series of height and mass measurements to least counts of 0.005 *cm* and 0.005 *kg*, respectively, by reproducible anthopometrists following laid-down, standardized protocols — child barefoot, stripped to short underpants, elbows and knees not flexed, instructed to inhale completely. These measurements are fed in software to generate profiles for each checkup from the first to the most-recent using 'Extended CDC Growth Charts and Tables', which have heights and masses listed for extreme percentiles — 0.01^{th}, 0.1^{th} and 1^{st} as well as 99^{th}, 99.9^{th} and 99.99^{th}, in addition to entries between 3^{rd} and 97^{th} percentiles. Values of height and mass percentiles at age of the most-recent checkup as well as the reference percentile are substituted in mathematical framework of the vector model to draw the desired percentile trajectories. Height- and mass-month-wise targets are determined by reading off values, where these trajectories intersect with lines parallel to the percentile (vertical) axis. These lines cross the age (horizontal) axis at the ages for which targets are proposed.

Keywords: Diet and exercise plans, *estimated-adult BMI*, height- and mass-percentile trajectories, lifestyle adjustment, month-wise height and mass management, optimal mass, parabolic curve

LIST OF ABBREVIATIONS

*cm***:** centimeter(s) • *m***:** meter(s) • *ft***:** foot(feet) • *in***:** inch(es) • *lb***:** pound(s) • *oz***:** ounce(s) • *kg***:** kilogram(s)

BMI: **B**ody-**M**ass **I**ndex

CDC: **C**enters for **D**isease **C**ontrol and Prevention, Atlanta, GA, United States http://www.cdc.gov

MP: **M**id-**P**arental

NGDS: **N**ational **G**rowth and **D**evelopmental **S**tandards for the Pakistani Children http://ngds-ku.org

SGPP: **S**ibling **G**rowth **P**ilot **P**roject — a subproject of the NGDS Pilot Project

INTRODUCTION

The American Medical Association declared obesity as a disease in 2013. The pandemic development of this condition in youngsters, which may be linked to serious psychological, physical and social consequences in a cause of concern for all. The first lady of United States, **Her Excellency, Michelle Obama** declared childhood obesity an epidemic for her country. The disease is contributing, significantly, to adult obesity, diabetes as well as noncommunicable diseases. Hence, it becomes important to detect the problem early so as to plan and implement efficient and effective intervention strategies. There is a dire need for a rigorous mathematical modeling to take up this issue.

This paper improves upon earlier models of childhood obesity, put forward by our group during the last three years, so that manageable targets for picking up height and gaining/reducing mass (weight) are generated, which are based on the notion that desired height/mass may not be achieved within a short span of 6 *months*, but smoothly attained till the child reaches 10^{th} birthday, when the set goal is reached as both height and mass percentiles trajectories tangentially approach reference-percentile line.

CHILDHOOD OBESITY IN PERSPECTIVE

Obesity is recognized as a problem globally among youngsters (Ng *et al.*, 2014; Ogden *et al.*, 2014). The

[¶]Main contribution of PhD dissertation of the second author, registered from Department of Physics, University of Karachi, under supervision of the first author

[*]*Correspondence***:** Prof. Dr. S. Arif Kamal **(**http://ngds-ku.org/kamal), PhD (Mathematical Neuroscience), MA, Johns Hopkins, Baltimore, MD, United States; Director, SF-Growth-and-Imaging Laboratory and Project Director, the NGDS Pilot Project (http://ngds-ku.org); Ex-Chairman, Department of Health, Physical Education and Sports Sciences; Ex-Dean, Faculty of Science and Faculty of Engineering; Ex-Acting Vice Chancellor, University of Karachi (http://www.uok.edu.pk), Karachi 75270, Pakistan; *Telephone***:** +92 21 9926 1077

[$]MPhil, Mathematics; PhD Candidate, Department of Physics, University of Karachi, under supervision of the first author

September 4, 2013	1st-Generation Solution of Childhood Obesity (Kamal *et al.*, 2013*f*)
September 4, 2014	2nd-Generation Solution of Childhood Obesity (Kamal *et al.*, 2014*a*)
June 01, 2015	3rd-Generation Solution of Childhood Obesity (Kamal, 2015*a*)
February 13, 2016	4th-Generation Solution of Childhood Obesity (Kamal *et al.*, 2016*a*)

Fig. 1. Solutions of childhood-obesity problem proposed by the Anthromathematics Group, University of Karachi — pictures of boy first appeared in Kamal (2015*c*)

incidence of childhood obesity has risen up in almost all parts of the world (Silventoinen *et al.*, 2010). This trend may be contributed by socioeconomic disparities (Stamatakis *et al.*, 2010) as well as ethnic inequalities (Pedersen *et al.*, 2016). The disease is contributing, significantly, to adult obesity, diabetes as well as non-communicable diseases (Whitaker *et al.*, 1997). A number of complications in adulthood may be related to childhood obesity (Bibbins-Domingo *et al.*, 2007), in particular, adult cardiovascular disease (Baker *et al.*, 2007; Lloyd *et al.*, 2010) as well as serious consequences in psychological (Hunsberger *et al.*, 2016), physical and social domains (Ludwig, 2007; Olds *et al.*, 2010).

The obesity epidemic spread, not only, in the developed countries, but also, as a pandemic in rest of the world with the exception of those areas, which have food shortages as well as famines (Sørensen, 2009). Miqueleiz *et al.* (2014; 2016) studied trend of childhood obesity in Spain. S. Z. Yanovski and J. A. Yanovski (2011) discussed philosophical issues of obesity prevalence in the United States. Han *et al.* (2010) are of the opinion that there is a need of reassessment of intake of calories and recommendation of physical activities so that there is an improved quantification at a population level because of sedentary lifestyles of youngsters these days. Further, community beliefs play a very important role in enlisting help of families (Aslam *et al.*, 2013; Covic *et al.*, 2007). There is a need to understand genetics of childhood obesity (Zhao and Grant, 2011) before looking at management approaches (Miller and Silverstein, 2007).

Once the condition has been recognized as a disease (Kyle *et al.*, 2016), there have been attempts towards solutions of childhood obesity (Finegood et al., 2010; Robinson and Sirard, 2005). During 2013-2016 our group proposed 1st- to 4th-generation mathematical-physical solutions of childhood obesity (Figure 1). The 4th-generation solution is elaborated with all possible cases worked out using clinical data.

Other than the NGDS (National Growth and Developmental Standards for the Pakistani Children) Pilot Project (described below), there are a few anthropometric studies in Pakistan, which determined obesity and malnutrition in school children (Ramzan *et al.*, 2008; Shafqat *et al.*, 2013).

THE NGDS PILOT PROJECT

Project Protocols

The NGDS Pilot Project started in 1998 in 4 representative schools (one civilian and one each operated by the Armed Forces of Pakistan — the Pakis-tan Air Force, the Pakistan Army and the Pakistan Navy) after the proposal went through 'Institutional Review Process' (Kamal *et al.*, 2002). A subproject of the NGDS Pilot Project, named as SGPP (Sibling Growth Pilot Project) served families, who came to SF-Growth-and-Imaging Laboratory along with their 5-10-*year*-old children, for check-ups, which were conducted giving due regard to participants' comfort, confidentiality, dignity, privacy and safety.

Anthropometric Techniques and Examinations

Children were screened for factors, which may contribute to growth abnormalities, including anemia, trunk deformities, in particular, scoliosis, lordosis and kyphosis (Kamal *et al.*, 2015*a*; 2016*d*). Their gaits were observed to screen for toes outward/inward as well as presence of spastic gait (Kamal *et al.*, 2016*b*). Heights, *h*, and masses, μ, were measured by trained anthropometrists, with documented reproducibility, as per protocols given in (Kamal, 2006*b*) to least counts of 0.1 *cm* (1998-2011, setsquare); 0.01 *cm* (2012-2015, Vernier scale); 0.005 *cm* (since early 2016, enhanced-Vernier scale) and 0.5 *kg* (1998-2011, bathroom scale); 0.01 *kg* (2012-2015, modified-beam scale); 0.005 *kg* (since early 2016, enhanced-beam scale), respectively, before noon, as children were 1-1.5 *cm* taller in the morning as compared to the afternoon. Children were required to be barefoot and undress completely except briefs or

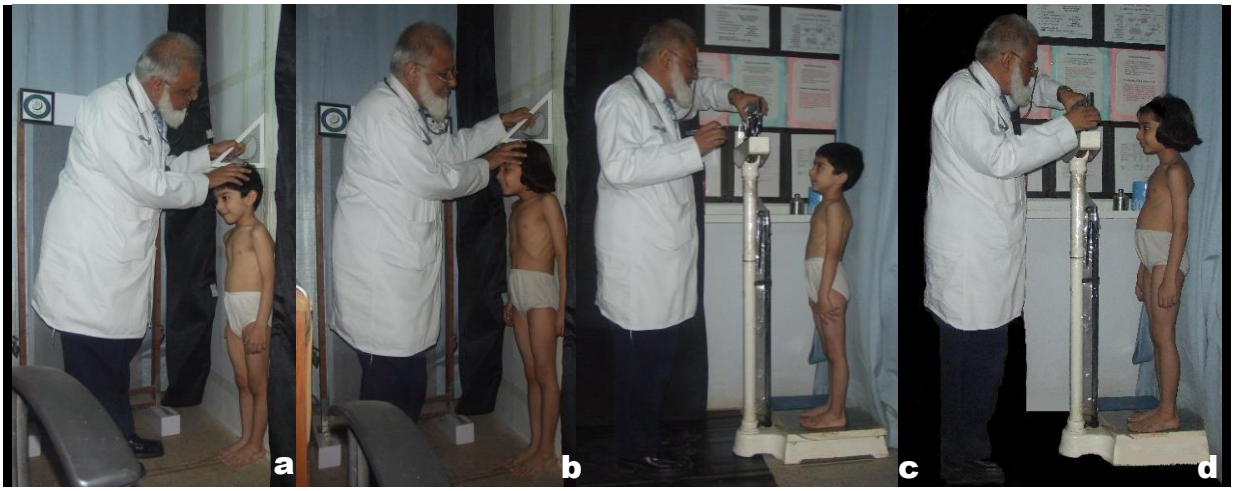

Fig. 2*a-d*. (*a*, *b*) Height and (*c*, *d*) mass measurements of a boy (Q. P.**:** SGPP-KHI-20081031-01/03, age 5 *years* 1 *month* 9 *days*) and a girl (E. P.**:** SGPP-KHI-20081031-01/02, age 7 *years* 11 *months*) in SF-Growth-and-Imaging Laboratory; ideal candidates for application of the vector model — (*a*), (*c*) first appeared in Kamal *et al.* (2015), whereas (*b*), (*d*) appeared in Kamal and Jamil (2014); the two sets printed in the same journal

panties for examinations and anthropometry. As the children were stripped, examiners were able to ascertain standard posture (knees and elbows not flexed, heels or toes not lifted) as well as complete inhaling during measurements (Figures 2*a-d*). Parents were measured barefooted and bareheaded, dressed in minimal indoor clothing.

GROWTH-AND-OBESITY ROADMAPS

Growth-and-Obesity Profiles

'Linear interpolation' was used to determine percentiles of mid-parental (target) height, P_{MP}, as well as army-cutoff height, P_{AC}, for a boy or a girl (Figure 2*e*) from age-20-height values of 'Extended CDC (Centers for Disease Control and Prevention) Growth Charts and Tables' (Kamal and Jamil, 2014). P_{MP}, is the percentile corresponding to gender-specific-adult-mid-parental (target) height (in *cm*) given by $h_{\mathrm{MP}} = \frac{h_{\mathrm{F}} + h_{\mathrm{M}}}{2} \pm 6.5\,cm$, where h_{F} and h_{M} were heights of father and mother measured in *cm*; + sign taken for boys' target height; – for girl's target height (Tanner *et al.*, 1970). P_{AC}, is based on country-specific-army-cutoff heights. For Pakistani youth, these heights are fixed at 5 *ft* 4 *in* (162.56 *cm*) for males and 5 *ft* 2 *in* (157.48 *cm*) for females, which correspond to 3rd (2.718014592103645… to be exact) and 19th (19.35609323536863… to be exact) *percentiles*, respectively (Kamal *et al.*, 2016*c*). 'Box interpolation' was used to evaluate child's height (mass) percentile (Kamal *et al.*, 2011). Estimated-adult height (mass) was computed through 'linear interpolation' using age-20 values and *estimated-adult BMI* obtained. *BMI* (body-mass index) cannot be used directly for children; *BMI* charts are needed for interpretation (Kamal *et al.*, 2013*e*). Optimal mass was computed by following constant-age route to determine obesity profile. A similar procedure was used to evaluate current-age-mid-parental height. Algebraic status (pertaining-to-height), $STATUS_{\pm}(h)$, as well as algebraic status (pertaining-to-mass), $STATUS_{\pm}(\mu)$, computed using ($h_{\mathrm{CA\text{-}MP}}$ is the adult-mid-parental height back extrapolated to current age)

Nomenclature	Depends on Child's Height	Depends on Parents' Heights	Based on Country-Wide Standards	Corresponding Percentile
Estimated-Adult Height, $h_{\mathrm{estimated\text{-}adult}}$	Yes	No	No	$P(h)$
Adult-Mid-Parental (Target) Height, h_{MP}	No	Yes	No	P_{MP}
Army-Cutoff Height, h_{AC}	No	No	Yes	P_{AC}

Fig. 2*e*. Heights used to determine reference height for the purpose of generating Growth-and-Obesity Roadmaps

Nomenclature	Represented by	Mathematical Expression	References
Body-Mass Index	*BMI*	μ/h^2	Keys *et al.* (1972)
Estimated-Adult Body-Mass index	$BMI_{\text{estimated-adult}}$	$\mu_{\text{estimated-adult}}/h_{\text{estimated-adult}}^2$	Kamal and Jamil (2012)
Optimal Mass	μ_{opt}	$P(\mu_{\text{opt}}) = P(h)$	Kamal *et al.* (2004; 2011*a*)

Fig. 2*f*. Measured heights and masses used to indicate childhood and adult obesity through different indicators

$$STATUS_{\pm}(h) = 100\frac{h - h_{\text{CA-MP}}}{h_{\text{CA-MP}}}\%;\ STATUS_{\pm}(\mu) = 100\frac{\mu - \mu_{\text{opt}}}{\mu_{\text{opt}}}\%.$$

Qualitative statuses were determined from algebraic statuses using recipes given in Tables $5_{\text{VECTOR}}a, b$ of Additional File 3. Note that the child was wasted if $P(\mu) < P(h)$ and obese if $P(\mu) > P(h)$. Figure 2*f* lists various indicators of childhood obesity. Sums of percentiles of heights and masses were used to assign builds (Figures 2*g*-*j*). The child was

Classification	Percentile Range	Dominating Function	Suitable for
Small	$0 \le P(h) + P(\mu) < 50$	Brain	Planning, development and intellectual work
Medium	$50 \le P(h) + P(\mu) < 150$	Brain and body functions equally contributing	May adapt to tasks dominating brain or body
Big	$150 \le P(h) + P(\mu) < 200$	Body	Assignments involving speed and strength

Fig. 2*g*-*j*. Measured heights and masses employed to determine percentiles used in (*g*) assigning build of a child; examples of children of (*h*) small build, E. P.**:** SGPP-KHI-20081031-01/02, age 7 *years* 11 *months*, $P(h) + P(\mu) = 8.08$, (*i*) medium build, R. Z. A. Q.**:** SGPP-KHI-20080423-01/02, age 4 *years* 3 *months* 9 *days*, $P(h) + P(\mu) = 54.29$ and (*j*) big build, R. Z. A.**:** SGPP-KHI-200910-01/02, age 3 *years* 4 *months* 11 *days*, $P(h) + P(\mu) = 161.13$ — pictures of children first appeared in Kamal and Khan (2015)

identified as acutely malnourished if $P(h) < 3$ and $P(\mu) < 3$ (Kamal and Jamil, 2014), severity given by (Kamal, 2015*b*)

$$\textit{Severity of Acute Malnutrition} = 100\left(1 - \frac{P(h) + P(\mu)}{6}\right)\%$$

Other nutritional statuses were classified on the basis algebraic statuses (Figure 2*k*). To compute Parents' Obesity Profiles, height and mass percentiles of mother and father were computed using 'linear interpolation' from lesser and

Classification	Description	$P(h)$, $STATUS_{\pm}(h)$	$P(\mu)$, $STATUS_{\pm}(\mu)$
Acute Malnutrition	Severe Stunting + Severe Wasting	$P(h) < 3$	$P(\mu) < 3$
Under-Nutrition	Stunting + Wasting	$STATUS_{\pm}(h) < 0$	$STATUS_{\pm}(\mu) < 0$
Over-Nutrition	Tallness + Obesity	$STATUS_{\pm}(h) > 0$	$STATUS_{\pm}(\mu) > 0$
Energy-Channelization I	Tallness + Wasting	$STATUS_{\pm}(h) > 0$	$STATUS_{\pm}(\mu) < 0$
Energy-Channelization II	Stunting + Obesity	$STATUS_{\pm}(h) < 0$	$STATUS_{\pm}(\mu) > 0$
Energy-Channelization III	Puberty-Induced Energy-Channelization	Height gain levels off	Below waist fat & mass gain

Fig. 2*k*. Measured heights and masses used to classify nutritional status (coördinate-plane and Venn-diagrammic representations in Figures $3_{\text{VECTOR}}a, b$, Additional File 3, page 9) — acute malnutrition is the limiting case of under-nutrition; energy-channelization III the limiting case of over-nutrition (Kamal, 2015*b*)

greater age-20 values of heights and masses read from extended-gender-specific tables.

Growth-and-Obesity Moving-Profiles

For the purpose of computing 'Growth-and-Obesity Moving-Profiles' the above model (Kamal *et al.*, 2011) was modified. The word 'moving profile' was borrowed from 'moving average', a terminology in statistics, and propagated the concept from the 'snap shot' health status to a 'time series' giving 6-*month* average (Kamal *et al.*, 2004; 2014*b*). The doctors are more interested in such indicators these days. An example could be cited from blood work of a patient. The blood glucose, fasting and random, is not as reliable as HBA_1C.

Growth-and-Obesity Scalar-Roadmaps

To construct 'Growth-and-Obesity Scalar-Roadmaps' (Kamal, 2015*a*; *b*; Kamal *et al.*, 2014*a*; 2015*b*) one needed more than one profile (each representing one checkup). Percentile of reference height, P_{ref}, was the maximum of 3 percentiles**:** measured-height percentile, $P(h)$; gender-specific-army-cutoff-height percentile, P_{AC}; gender-specific-mid-parental-height percentile, P_{MP} (Figure 2*e*). Optimal mass, 6 *months* after the date of current checkup, was determined on the basis of estimated-reference height (6 *month* ahead in time scale). Recommendations to pick up height and put on or shed off mass (weight) were generated from the most-recent profile. Difference of measured height (current value) and reference height (after 6 *months*) was considered as guideline to set short-term goals to gain height within the next 6 *months*. Similarly, difference of measured mass (current value) and optimal mass (after 6 *months*) was considered as guideline to set short-term goals to gain (lose) mass within the next 6 *months*, if the value was negative (positive). Monthly recommendations to gain height or acquire (reduce) mass were prepared, taking care of the principle that a child should not be required to lose more than 10 *kg* within the next 6 *months*, in order to avoid any adverse health effects from a rapid loss of mass. 6 recommendations (on date of checkup of each successive month) to achieve specific values of height were obtained by adding monthly recommendation (to pick up height) multiplied by a factor (1 to 6, representing the number of successive month) to measured height (in *cm*) and converted into *ft* and *in*. In a similar manner, 6 recommendations (on date of checkup of each successive month) to achieve specific values of mass (weight) were obtained by adding monthly recommendation (to put on or shed off mass) multiplied by a factor (1 to 6, representing the number of successive month) to measured mass (current value in *kg*) and converted into weight (in *lb* and *oz*). Mother (father) was suggested to gain mass (weight) corresponding to the difference obtained by subtracting net mass from optimal mass, if the value of former was lesser than that of later. In case, value of net mass was greater than optimal mass, father was advised to shed off mass corresponding to that difference, provided the value was under-10 *kg*, otherwise he should reduce 10 *kg* within the next 6 *months*. For mothers (married at the time of check up or divorced/widowed in the near past), the recommendation to reduce mass was computed by adding 5 *kg* to mass, to account for possible pregnancy and the associated mass of fetus. From these differences, month-wise as well as date-wise recommendations to manage mass (weight) were prepared.

Need for Enhancement of the Scalar-Roadmap Model

The scalar-roadmap model attempts to achieve desired height and mass within the next 6 *months*. The short-term targets are generated by diving the 6-*month*-achievement goals into equal parts, which is not the way a child gains height during the childhood phase, according to ICP model (Karlberg, 1987). More-recent modeling of human growth from fetal stage to adolescent phase (Nirop *et al.*, 2016) should be looked into before devising a desired course-of-action.

Growth-and-Obesity Vector-Roadmaps

'Growth-and-Obesity Vector-Roadmaps' are identical to 'Growth-and-Obesity Scalar-Roadmaps' in the age range during which actual checkups are performed. However, they differ in height- and mass-management goals. These goals are generated by fitting a parabolic curve to height and mass desired trajectories originating at the age of most-recent checkup and terminating at the reference age, taken as 10 *years*, lower limit of the peripubertal phase (defined as the age, when the child is about to enter puberty, marked by rate of height gain approaching zero), in such a way that slope of desired trajectory matches with the reference trajectory (both slopes vanish, reference trajectory becomes tangent). Such a mechanism generates softer targets, which are easier to achieve. The end result is that correction is achieved by the end-of-childhood phase. The motivation being**:**

- The middle path between *too much, too soon* (achieving all corrections within 6 months; exerting un-necessary stress on child's skeleton and muscles) and *too little, too late* (waiting till the end of growth period, which is 19 *years* for girls and 21 years for boys to achieve complete correction; heights and masses are so much off from reference values that control action is not effective — every control law is valid within the given domain;

outside this domain controller ceases to work) is followed. This is, also, the according to the recommendations of multi-stage stage-Q system (Kamal and Mirza, 2006), where section-wise corrections are recommended — achieving corrections at the end of childhood phase and not waiting till a boy or girl stops gaining height.

- Maintenance of optimal mass is possible when values as well as slopes match for percentile curves of height and mass — dynamical-system approach (Kamal, 2015*a*). This concept has been taken from astrodynamical application in the context of a control law for satellite-launch vehicles — matching of positions and velocities as proposed in multi-stage (Kamal and Mirza, 2006) and extended-Q systems (Kamal, 2014).
- Matching of values and slopes of percentiles would ensure that height gain (tissue synthesis) remains a quasi-static process.
- A parabolic curve for desired trajectory of height is fitted as child's height follows a parabolic curve according to ICP model (Karlberg, 1987; Kamal and Jamil, 2012). A parabolic curve for desired trajectory of mass is fitted because optimal mass is defined by the condition that percentile of optimal mass matches with percentile of height. Hence, the functional relationship of desired trajectory of mass should replicate functional relationship of desired trajectory of height.

METHOD OF CONSTRUCING GROWTH-AND-OBESITY VECTOR-ROADMAPS

To understand the method of construction of 'Growth-and-Obesity Vector-Roadmaps', one needs to learn concepts of guidance, navigation and control as they are used in astrodynamics and space-flight dynamics.

Concepts of Navigation, Guidance and Control

'Navigation' is the answer to question, where the spacecraft is currently located in space, 'guidance', in which orbit the spacecraft is desired to be and 'control', what action is needed to bring the spacecraft to the desired orbit (Kamal, 2002). In the context of Growth-and-Obesity Roadmaps, 'navigational trajectory' represents the curve obtained from percentiles of actual checkups, 'guidance trajectory' the desired curve (actually, a straight line based on percentile of reference height) to be achieved at a pre-decided time (end of childhood phase in the context of Vector-Roadmap) and interventions involving lifestyle adjustment, diet and exercise plans through which height (mass) is 'controlled' (Kamal and Khan, 2013; Kamal *et al.*, 2013*a*, *b*, *e*).

Another concept, which is applicable in the mathematical framework of Growth-and-Obesity vector Roadmap is 'explicit guidance scheme' (Kamal, 2006*a*). As additional checkup data become available, fresh trajectories are drawn originating at the age of most-recent checkup to smoothly approach reference percentile at reference age.

Generation of Navigational Trajectories

The navigational trajectories for height and mass percentiles are obtained by fitting freehand curves to the respective height- and mass-percentile values, $P = P(h, A)$ and $P = P(\mu, A)$, respectively — A representing age (in *years*, expressed in decimal form) at the time of checkup, the domain (defined as set of all values permissible for the independent variable, which is A in our case) for navigational trajectory being $A_{\text{Enrolment}} \le A \le A_0$ ($A_{\text{Enrolment}}$ denoting age of child at the time of enrolment, which, also, coincides with the date of the first checkup, A_0 denoting age at the most-recent checkup, when control action needs to be initiated). The percentiles are computed from heights and masses of children recorded during checkups. These trajectories are shown as solid-blue curve (navigational trajectory representing height) and solid-maroon curve (navigational trajectory representing mass) in Figures 3*a-f*. The navigational trajectories are common for both Scalar- and Vector-Roadmaps.

Generation of Guidance Trajectory

Let $P(h, A_0)$ represent percentile of height, which was measured during the most-recent checkup. As mentioned in the section 'Growth-and-Obesity Scalar-Roadmaps', the percentile of reference height, P_{ref}, may be expressed as $P_{\text{ref}} = max(P(h, A_0), P_{\text{AC}}, P_{\text{MP}})$. The guidance trajectory, therefore, takes the form, $P = P_{\text{ref}}$, shown as green-dashed line, the domain for guidance trajectory being $A_0 \le A \le 10$ (10 *years* is the age, when control action is terminated as height and mass percentiles take up the value of reference percentile in such a way that reference line becomes tangent to desired-percentile trajectory, representing heights or masses, at the terminal point).

Generation of Control-Action Trajectories

A parabolic curve (appendix A) is fitted to generate trajectory of desired course-of action, when is passing through A_0, the age, when control action is initiated and smoothly touches the guidance trajectory, $P = P_{\text{ref}}$, at the

Table 1. Different cases to be dealt in the vector model
(condition for the validity of this model, $A_0 < 9.5\,years$)

Case No.	*Conditions for Percentiles*		*Worked Examples*	
	$P(h, A_0)$	$P(\mu, A_0)$	*Table*	*Figure*
1	$P(h, A_0) = P_{\text{ref}}$	$P(\mu, A_0) = P_{\text{ref}}$	3*a*	3*a*
2	$P(h, A_0) = P_{\text{ref}}$	$P(\mu, A_0) < P_{\text{ref}}$	3*b*	3*b*
3	$P(h, A_0) = P_{\text{ref}}$	$P(\mu, A_0) > P_{\text{ref}}$	3*c*	3*c*
4	$P(h, A_0) < P_{\text{ref}}$	$P(\mu, A_0) = P_{\text{ref}}$	3*d*	3*d*
5	$P(h, A_0) < P_{\text{ref}}$	$P(\mu, A_0) < P_{\text{ref}}$	3*e*	3*e*
6	$P(h, A_0) < P_{\text{ref}}$	$P(\mu, A_0) > P_{\text{ref}}$	3*f*	3*f*

vertex $(10, P_{\text{ref}})$, the guidance trajectory, therefore, becomes tangent to parabola at this point. Percentile of height, *h*, as a function of age, *A*, may, then, be expressed as**:**

(1*a*) $$P(h, A) = P_{\text{ref}}\,;\ \text{if}\ \ P(h, A_0) = P_{\text{ref}}$$

(1*b*) $$P(h, A) = P_{\text{ref}} - \left(P_{\text{ref}} - P(h, A_0)\right)\left(\frac{A-10}{A_0-10}\right)^2;\ \text{otherwise}$$

Similar equations are written for percentile of mass, μ**:**

(2*a*) $$P(\mu, A) = P_{\text{ref}}\,;\ \text{if}\ P(\mu, A_0) = P_{\text{ref}}$$

(2*b*) $$P(\mu, A) = P_{\text{ref}} - \left(P_{\text{ref}} - P(\mu, A_0)\right)\left(\frac{A-10}{A_0-10}\right)^2;\ \text{otherwise}$$

where $P(\mu, A_0)$ represents percentile of mass at the most-recent checkup. The recommendations for height and mass are now generated by drawing lines parallel to percentile
Month-wise targets to pick-up height- and put-on/shed-off mass are determined by reading off values, where these desired course-of-action trajectories (dashed-blue and dashed-maroon curves, blue representing height trajectory, maroon mass trajectory) with lines parallel to the percentile axis, which is, actually, the vertical axis. These lines cross the age axis, which is, actually, the horizontal axis, at the ages for which targets are proposed. In the 'Results and Discussion' section, height and mass recommendations based on the vector model and the scalar model of the 6 possible cases (Table 1) are compared. The differences are obtained using the expressions, $\Delta h = h_{\text{Vector}} - h_{\text{Scalar}}$; $\Delta\mu = \mu_{\text{Vector}} - \mu_{\text{Scalar}}$. The subscript 'Vector' ('Scalar') denotes value generated from the vector (the scalar) model.

Sample Vector-Roadmap

Growth-and-Obesity Vector-Roadmap of L. G., who gained height rapidly because of her participation in spots, but severely lagged behind in mass put-on goals, is presented on the following page (Table 2).

RESULTS

Worked examples of cases 1-6 appear below, which were listed in Table 1. Predications based on the vector model and the scalar model (Kamal *et al.*, 2015*b*) are compared. Guidance, navigation and control-action trajectories are shown.

***Case* 1: T. J. (SGPP-KHI-20060412-01/01)**

This is the case $P(h, A_0) = P_{\text{ref}} \bullet P(\mu, A_0) = P_{\text{ref}}$. Although, $P(\mu, A_0)$ = 20.2555713826592 for T. J. at the age of 7.91506849315078 *years* corresponding to a mass 22.5 *kg*, we are using a value matching reference percentile to demonstrate the procedure — a larger difference in the first as well as the second recommendation is due to this manipulation. Desired trajectories are given by

(3*a*) $$P(h, A) = 20.9034295857707;\ P(\mu, A) = 20.9034295857707$$

Table 2. Growth-and-Obesity Vector-Roadmap of L. G. (SGPP-KHI-20131021-02/01), a girl participating in gymnastics, information optimized, also see Table 3*b* and Figure 3*b*, comprehensive report uploaded separately as Additional File 3 (http://www.ngds-ku.org/Papers/J43/Additional_File_3.pdf) — compare with Scalar-Roadmap**:** Additional File 2 (http://www.ngds-ku.org/Papers/J43/Additional_File_2.pdf)[∀]

*Gender***:** Female ♀ • *Date of Birth* (*year-month-day*)**:** 2007-08-15 • *Army-Cutoff Height***:** 157.48 *cm* (19.36^{P})[§]
*Father's Height***:** ♂ 167.16 *cm* • *Mother's Height***:** ♀ 160.16 *cm* • *Target Height***:** 157.16 *cm* (18.14^{P})

Checkup	*1st*	*2nd*	*3rd*	
Photograph				
Scanned Signatures	*LG*	*LG*	*LG*	*LG*
Class and Section	II-B	II-B	III-B	III-B
Date of Checkup (*year-month-day*)	2014-11-22	2015-02-28	2015-08-22	2016-03-26
Age (*year-month-day*)	07-03-07	07-05-23	08-00-07	08-07-11
Age (*decimal year*)	7.27	7.54	8.02	8.61
Dress Code	0/0.5	0/0.5	0/0.5	0/0.5
Behavior Code	0	0	0	0
Height, *h* (*cm*)	126.96	139.92	143.51	147.255
Height (*ft-in*)	4 *ft* 1.98 *in*	4 *ft* 7.09 *in*	4 *ft* 8.50 *in*	4 *ft* 9.97 *in*
Percentile-for-Height, *P*(*h*)	74.37	99.01	99.06	99.06
Estimated-Adult Height (*cm*)	167.59	180.03	180.39	180.37
Estimated-Adult Height (*ft–in*)	**5 *ft* 5.98 *in***	**5 *ft* 10.88 *in***	**5 *ft* 11.02 *in***	**5 *ft* 11.01 *in***
Current-Age-MP[⊇] Height (*cm*)	118.00	119.59	122.25	125.27
Δ Height *w. r. t.* Current-Age-MP Height (*cm*)	+8.96	+20.33	+21.26	+21.99
Algebraic Status (pertaining-to-height)	+7.59%	+17.00%	+17.39%	+17.55%
Qualitative Status (pertaining-to-height)	**1st-Deg Tall**	**2nd-Deg Tall**	**2nd-Deg Tall**	**2nd-Deg Tall**
Current-Age-Army-Cutoff Height (*cm*)	118.32	119.86	122.53	125.56
Δ Height *w. r. t.* Army-Cutoff Height (*cm*)	+8.64	+20.06	+20.98	+21.70
Reference Height (*cm*)	126.96	139.92	143.51	147.255
Percentile-for-Reference-Height	74.37	99.01	99.06	99.06
Gross Mass (*kg*)	23.66	25.69	28.21	29.975
Clothing Correction (*kg*)	0	0	0	0
Net Mass, μ (*kg*)	23.66	25.69	28.21	29.975
Net Weight (*lb-oz*)	52 *lb* 2.72 *oz*	56 *lb* 10.34 *oz*	62 *lb* 3.25 *oz*	66 *lb* 1.52 *oz*
Percentile-for-Net-Mass, $P(\mu)$	51.31	61.58	68.54	65.29
Estimated-Adult Mass (*kg*)	58.62	61.76	63.88	62.89
Estimated-Adult Weight (*lb–oz*)	129 *lb* 4.04 *oz*	136 *lb* 2.73 *oz*	143 *lb* 13.64 *oz*	138 *lb* 10.62 *oz*
Optimal Mass (*kg*)	26.37	39.12	43.33	47.55
Δ Mass-for-Height (*kg*)	–2.71	–13.43	–15.12	–17.58
Algebraic Status (pertaining-to-mass)	–10.28%	–34.33%	–34.90%	–36.96%
Qualitative Status (pertaining-to-mass)	**2nd-Deg Wasted**	**4th-Deg Wasted**	**4th-Deg Wasted**	**4th-Deg Wasted**
Cumulative-Scoliosis-Risk Weightage[®]	8.50	9.00	9.00	9.50
Estimated-Adult *BMI* (kg/m^2)	20.87	19.05	19.63	19.33
Nutritional Status	**EC I**	**EC I**	**EC I**	**EC I**
$P(h) + P(\mu)$	125.68	160.59	167.60	164.35
Build	**Medium**	**Big**	**Big**	**Big**

[∀]Color-coding of table entries is explained in Figure 3_{VECTOR},Tables 4_{VECTOR}*a-c* and 5_{VECTOR}*a*, *b* of Additional File 3.
[§]The superscript P stands for percentile.
[⊇]MP stands for mid-parental.
[®]Cumulative-Scoliosis-Risk Weightage (CSRW) is explained in Kamal *et al.* (2013*d*).

Table 3*a*. Comparison of T. J.'s month-wise-height and -mass (-weight) management generated from both models

Gender: Male ♀ • *Date of Birth* (*year-month-day*): 2003-04-20 • *Army-Cutoff Height*: 162.56 *cm* (2.72^{P})
Date of Last (*Fifth*) *Checkup* (*year-month-day*): 2011-03-20 • *Decimal Age*, A_0 = 7.915068493 *years*
Father's Height: ♂ 165.70 *cm* • *Mother's Height*: ♀ 155.73 *cm* • *Target Height*: 167.22 *cm* (9.35^{P})
P_{ref} = 20.9034295857707 • $P(h, A_0)$ = 20.9034295857707 • $P(\mu, A_0)$ = 20.9034295857707

Target Date	*Height* (*h*) *Target* (*cm*)			*Mass* (μ) *Target* (*kg*)		
	VECTOR MODEL	SCALAR MODEL	Δh	VECTOR MODEL	SCALAR MODEL	$\Delta\mu$
April 20, 2011	123.09	123.06	+0.03	22.76	22.71	+0.05
May 20, 2011	123.54	123.53	+0.01	22.96	22.91	+0.05
June 20, 2011	124.00	123.99	+0.01	23.15	23.12	+0.03
July 20, 2011	124.45	124.45	0	23.34	23.33	+0.01
August 20, 2011	124.91	124.91	0	23.54	23.54	0
September 20, 2011	125.38	125.38	0	23.74	23.74	0

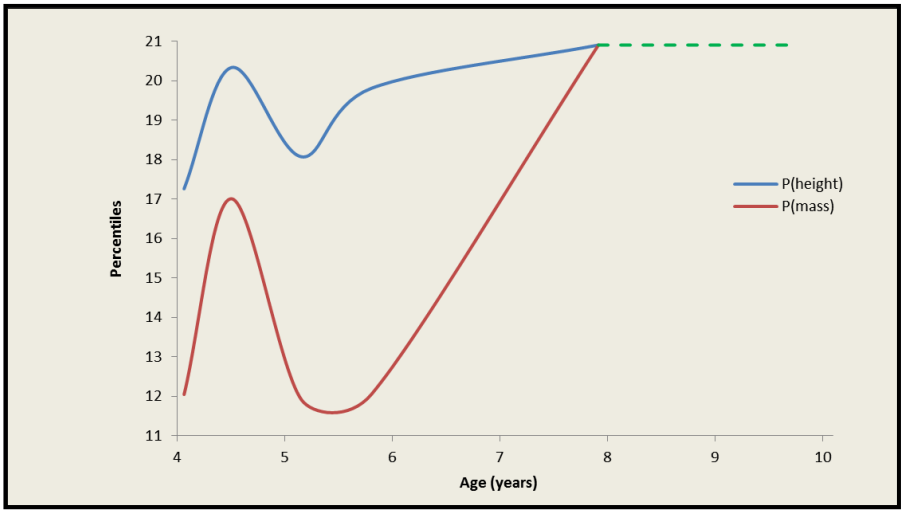

Fig. 3*a*. Time evolution of T. J.'s height and mass percentiles for his five checkups in the age range 4.06-7.92 *years* (navigational trajectory: solid curve), including the desired course-of-action (guidance trajectory: green-dashed line) and recommended intervention (control action: none)

Table 3*a* compares results generated from the vector model and the scalar model. Control action is needed neither for the height trajectory nor the mass trajectory (Figure 3*a*).

***Case* 2: L. G. (SGPP-KHI-20131021-02/01)**

This is the case $P(h, A_0) = P_{ref}$ • $P(\mu, A_0) < P_{ref}$. Table 3*b* compares results generated from the vector model and

Table 3*b*. Comparison of L. G.'s month-wise-height and -mass (-weight) management generated from both models

Gender: Female ♀ • *Date of Birth* (*year-month-day*): 2007-08-15 • *Army-Cutoff Height*: 157.48 *cm* (19.36^{P})
Date of Last (*Fourth*) *Checkup* (*year-month-day*): 2016-03-26 • *Decimal Age*, A_0 = 8.613054869 *years*
Father's Height: ♂ 167.16 *cm* • *Mother's Height*: ♀ 160.16 *cm* • *Target Height*: 157.16 *cm* (18.14^{P})
P_{ref} = 99.0559863 • $P(h, A_0)$ = 99.0559863 • $P(\mu, A_0)$ = 65.28545564

Target Date	*Height* (*h*) *Target* (*cm*)			*Mass* (μ) *Target* (*kg*)		
	VECTOR MODEL	SCALAR MODEL	Δh	VECTOR MODEL	SCALAR MODEL	$\Delta\mu$
April 26, 2016	147.77	147.76	+0.01	30.93	33.56	–2.63
May 26, 2016	148.28	148.29	–0.01	31.84	37.14	–5.30
June 26, 2016	148.79	148.80	–0.01	32.98	40.73	–7.75
July 26, 2016	149.30	149.32	–0.02	34.34	44.31	–9.97
August 26, 2016	149.82	149.83	–0.01	35.71	47.89	–12.18
September 26, 2016	150.35	150.35	0	37.04	51.48	–14.44

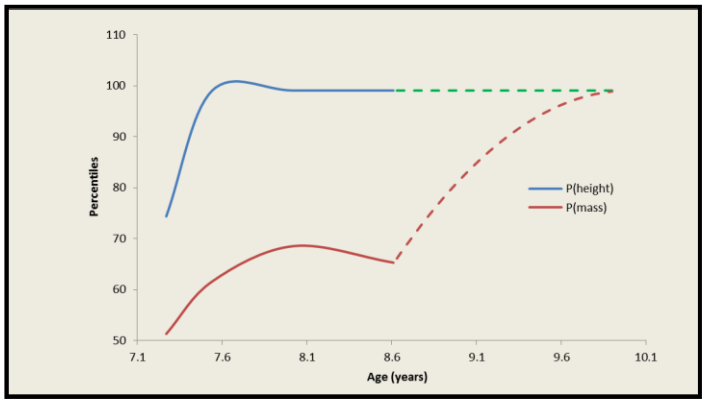

Fig. 3*b*. Time evolution of height and mass percentiles of L. G. for her four checkups in the age range 7.27-8.61 *years* (navigational trajectory**:** solid curve), including the desired course-of-action (guidance trajectory**:** green-dashed line) and recommended intervention (control action**:** none for height-percentile curve and maroon-dashed for mass-percentile curve)

the scalar model. Desired trajectories are given in Figure 3*b*; no control action needed for the height trajectory

(3*b*) $$P(h, A) = 99.0559863;\ P(\mu, A) = 99.0559863 - 17.5557474\,(A-10)^2$$

***Case* 3: Q. H. (NGDS-BLA-2010-4657/Z)**

This is the case $P(h, A_0) = P_{\text{ref}} \bullet P(\mu, A_0) > P_{\text{ref}}$, illustrated in Table 3*c* and Figure 3*c*. No control action is

Table 3*c*. Comparison of Q. H.'s month-wise-height and -mass (-weight) management generated from both models

*Gender***:** Female ♀ • *Date of Birth* (*year-month-day*)**:** 2006-01-12 • *Army-Cutoff Height***:** 157.48 *cm* (19.36^{P})
Date of Last (*Third*) *Checkup* (*year-month-day*)**:** 2013-06-12 • *Decimal Age*, A_0 = 7.4136986301 *years*
*Father's Height***:** ♂ 166.56 *cm* • *Mother's Height***:** ♀ 156.63 *cm* • *Target Height***:** 155.10 *cm* (10.29^{P})
P_{ref} = 37.68905385892 • $P(h, A_0)$ = 37.68905385892 • $P(\mu, A_0)$ = 45.2289783126694

Target Date	*Height* (*h*) *Target* (*cm*)			*Mass* (μ) *Target* (*kg*)		
	VECTOR MODEL	SCALAR MODEL	Δh	VECTOR MODEL	SCALAR MODEL	$\Delta\mu$
July 12, 2013	122.76	122.74	+0.02	23.63	23.56	+0.07
August 12, 2013	123.25	123.23	+0.02	23.82	23.66	+0.16
September 12, 2013	123.74	123.72	+0.02	24.02	23.77	+0.25
October 12, 2013	124.22	124.21	+0.01	24.21	23.88	+0.33
November 12, 2013	124.71	124.69	+0.02	24.41	23.99	+0.42
December 12, 2013	125.18	125.18	0	24.60	24.09	+0.51

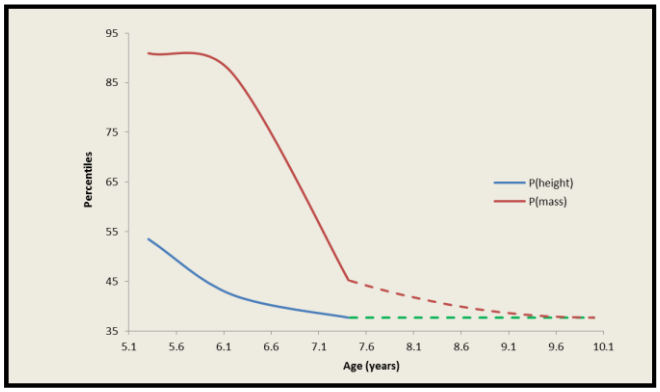

Fig. 3*c*. Time evolution of height and mass percentiles of Q. H. for her three checkups in the age range 5.31-7.41 *years* (navigational trajectory**:** solid curve), including the desired course-of-action (guidance trajectory**:** green-dashed line) and recommended intervention (control action**:** none for height-percentile curve and maroon-dashed for mass-percentile curve)

needed for the height trajectory. The desired trajectories are governed by the equations

(3*c*) $$P(h, A) = 37.68905385892;\ P(\mu, A) = 37.68905385892 + 1.1272201272\,(A-10)^2$$

***Case* 4: Z. I. R. (SGPP-KHI-20100908-01/04)**

This is the case $P(h, A_0) < P_{\text{ref}} \bullet P(\mu, A_0) = P_{\text{ref}}$, illustrated in Table 3*d* and Figure 3*d*. Although $P(\mu, A_0)$ = 17.8923302986379 for Z. I. R. at the age of 7.77868852459005 *years* corresponding to a mass 21.56 *kg*, we are using a value matching reference percentile to demonstrate the procedure — a larger difference in the first, the second as well as the third recommendation is due to this manipulation. The desired trajectories are governed by the equations

(3*d*) $$P(h, A) = 19.3560932353686 - 2.917446382\,(A-10)^2;\ P(\mu, A) = 19.3560932353686$$

As her mass matches with the reference mass, no control action is needed for mass trajectory.

Table 3*d*. Comparison of Z. I. R.'s month-wise-height and -mass (-weight) management generated from both models

Gender: Female ♀ • *Date of Birth* (*year-month-day*): 2004-03-29 • *Army-Cutoff Height*: 157.48 *cm* (19.36^{P})
Date of Last (*Second*) *Checkup* (*year-month-day*): 2012-01-08 • *Decimal Age*, A_0 = 7.7786885246 *years*
Father's Height: ♂ 164.02 *cm* • *Mother's Height*: ♀ 151.12 *cm* • *Target Height*: 151.07 *cm* (2.98^{P})
P_{ref} = 19.3560932353686 • $P(h, A_0)$ = 4.96075732220966 • $P(\mu, A_0)$ = 19.3560932353686

Target Date	*Height* (*h*) *Target* (*cm*)			*Mass* (μ) *Target* (*kg*)		
	VECTOR MODEL	SCALAR MODEL	Δh	VECTOR MODEL	SCALAR MODEL	$\Delta\mu$
February 8, 2012	118.01	118.25	–0.24	21.94	21.79	+0.15
March 8, 2012	118.82	119.37	–0.55	22.12	22.03	+0.09
April 8, 2012	119.65	120.49	–0.84	22.33	22.26	+0.07
May 8, 2012	120.43	121.61	–1.18	22.54	22.49	+0.05
June 8, 2012	121.22	122.73	–1.51	22.75	22.73	+0.02
July 8, 2012	121.86	123.85	–1.99	22.96	22.96	0

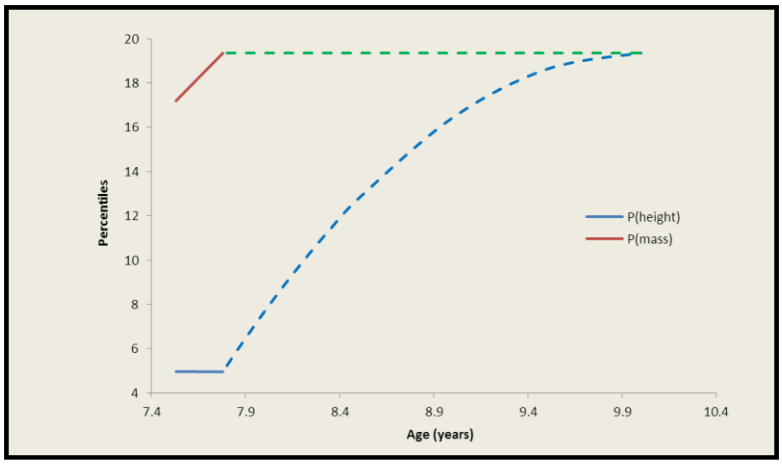

Fig. 3*d*. Time evolution of height and mass percentiles of Z. I. R. for her two checkups in the age range 7.53-7.78 *years* (navigational trajectory: solid curve), including the desired course-of-action (guidance trajectory: green-dashed line) and recommended intervention (control action: blue-dashed for height-percentile curve and none for mass-percentile curve)

***Case* 5: Hr. S. (SGPP-KHI-20110614-01/01; NGDS-BLA-2011-5085/H)**

This is the case $P(h, A_0) < P_{\text{ref}} \bullet P(\mu, A_0) < P_{\text{ref}}$. The desired trajectories are governed by the equations

(3*e*) $$P(h, A) = 36.4898701179913 - 7.001377713\,(A-10)^2;\ P(\mu, A) = 36.4898701179913 - 3.68386789\,(A-10)^2$$

Control action needs to be taken for both height and mass trajectories as both parameters are lagging behind the reference percentile.

Table 3*e*. Comparison of Hr. S.'s month-wise-height and -mass (-weight) management generated from both models

Gender: Female ♀ • *Date of Birth* (*year-month-day*): 2005-04-10 • *Army-Cutoff Height*: 157.48 *cm* (19.36^{P})
Date of Last (*Third*) *Checkup* (*year-month-day*): 2013-11-21 • *Decimal Age*, A_0 = 8.6164383562 *years*
Father's Height: ♂ 172.01 *cm* • *Mother's Height*: ♀ 162.94 *cm* • *Target Height*: 160.975 *cm* (36.49^{P})
P_{ref} = 36.4898701179913 • $P(h, A_0)$ = 23.0875330849453 • $P(\mu, A_0)$ = 10.2956242358146

Target Date	*Height* (*h*) *Target* (*cm*)			*Mass* (μ) *Target* (*kg*)		
	VECTOR MODEL	SCALAR MODEL	Δh	VECTOR MODEL	SCALAR MODEL	$\Delta\mu$
December 21, 2013	127.22	127.25	–0.03	23.18	23.38	–0.20
January 21, 2014	127.90	128.04	–0.14	23.83	24.23	–0.40
February 21, 2014	128.55	128.84	–0.29	24.48	25.08	–0.60
March 21, 2014	129.12	129.64	–0.52	25.04	25.94	–0.90
April 21, 2014	129.74	130.43	–0.69	25.65	26.79	–1.14
May 21, 2014	130.31	131.23	–0.92	26.22	27.64	–1.42

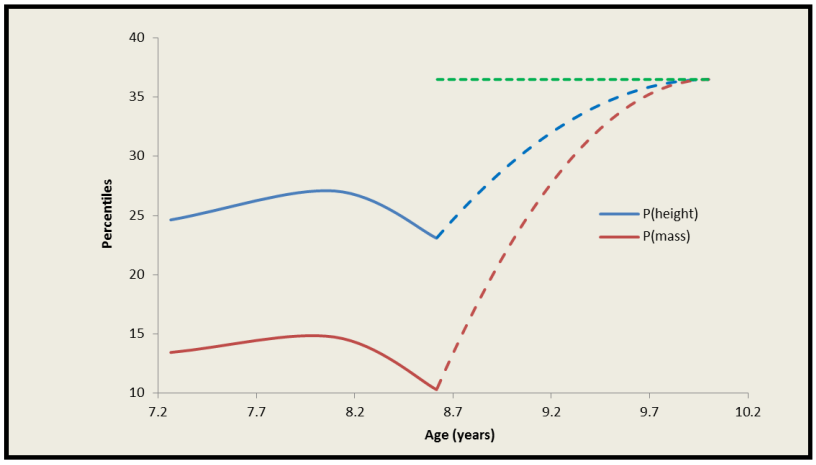

Fig. 3*e*. Time evolution of height and mass percentiles of Hr. S. for her three checkups in the age range 7.26-8.62 *years* (navigational trajectory: solid curve), including the desired course-of-action (guidance trajectory: green-dashed line) and recommended intervention (control action: blue-dashed for height-percentile curve and maroon-dashed for mass-percentile curve)

Table 3*e* and Figure 3*e* illustrate these cases.

***Case* 6: M. E. (SGPP-KHI-20100421-03/01)**

This is the case $P(h, A_0) < P_{ref}$ • $P(\mu, A_0) > P_{ref}$, illustrated in Table 3*f*. Control action needs to be taken for both height and mass trajectories as the height percentile is lagging behind and the mass percentile exceeds the reference percentile.

Table 3*f*. Comparison of M. E.'s month-wise-height and -mass (-weight) management generated from both models

Gender: Female ♀ • *Date of Birth* (*year-month-day*): 2002-09-23 • *Army-Cutoff Height*: 157.48 *cm* (19.36^{P})
Date of Last (*Second*) *Checkup* (*year-month-day*): 2011-11-13 • *Decimal Age*, A_0 = 9.139726027 *years*
Father's Height: ♂ 166.80 *cm* • *Mother's Height*: ♀ 171.00 *cm* • *Target Height*: 162.40 *cm* (44.64^{P})
P_{ref} = 44.6360788438672 • $P(h, A_0)$ = 34.3311209453783 • $P(\mu, A_0)$ = 63.4213131272544

Target Date	*Height* (*h*) *Target* (*cm*)			*Mass* (μ) *Target* (*kg*)		
	VECTOR MODEL	SCALAR MODEL	Δh	VECTOR MODEL	SCALAR MODEL	$\Delta\mu$
December 13, 2011	131.71	131.70	+0.01	31.55	31.98	–0.43
January 13, 2012	132.42	132.40	+0.02	31.34	32.17	–0.83
February l3, 2012	133.09	133.10	–0.01	31.18	32.36	–1.18
March 13, 2012	133.69	133.80	–0.11	31.08	32.55	–1.47
April 13, 2012	134.31	134.50	–0.19	31.05	32.74	–1.69
May 13, 2012	134.88	135.20	–0.32	30.40	32.93	–2.53

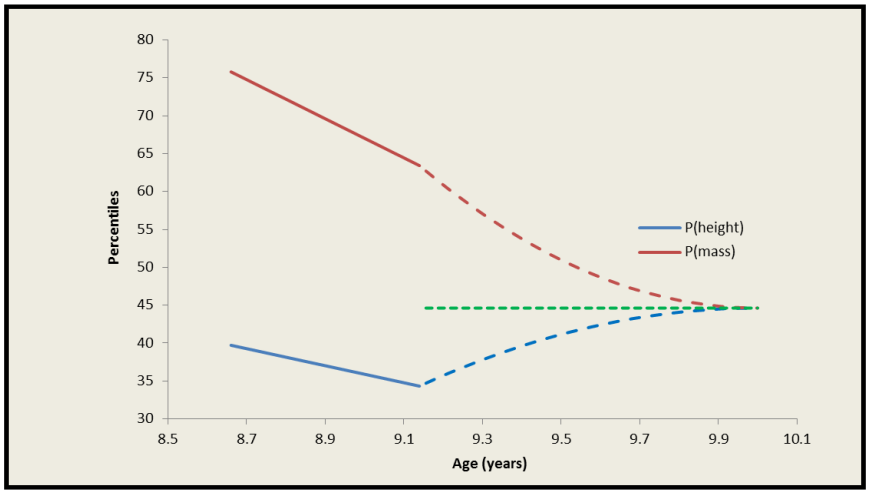

Fig. 3*f*. Time evolution height and mass percentiles of M. E. for her two checkups in the age range 8.66-9.14 *years* (navigational trajectory**:** solid curve), including the desired course-of-action (guidance trajectory**:** green-dashed line) and recommended intervention (control action**:** blue-dashed for height-percentile curve and maroon-dashed for mass-percentile curve)

The desired trajectories (Figure 3*f*) are governed by the equations

(3*f*) $P(h, A) = 44.6360788438672 - 13.92427700\,(A-10)^2$; $P(\mu, A) = 44.6360788438672 + 25.38300575\,(A-10)^2$

DISCUSSION

The vector model, presented in this work, has the following advantages over the scalar model proposed last year (Kamal *et al.*, 2015*b*)**:**

- Report has been divided into different parts, highlighted by color-codes for easy comprehension**:** vital statistics — yellow, height related — blue, mass related — pink, height and mass combined together — green (Table 2)
- Information in the report has been optimized, so that parents could note key findings and recommendations without going through unnecessary details.
- Availability of SMART (specific-manageable-attainable-realistic-time-bound) short-term and long-term goals to maintain optimal mass (weight) as well as attain desired height by the age of 10 *years*, instead of pushing the child to achieve a lot in a short span of time, which may bring about tendencies of anorexia nervosa or, in extreme cases, bulimia nervosa.
- Guidelines provided to the children and their parents for lifestyle adjustment, diet and exercise plans. optimal-mass management has been identified as optimal solution of diet, exercise and lifestyle adjustment — opti-mization approach (Kamal, 2015*b*; Kamal *et al.*, 2013*e*; 2014*b*). Table 4 on next page summarizes these ideas.

FUTURE DIRECTIONS

Future work in this area should focus on modeling of child growth and obesity, growth charts and enhancement of anthropometrric instruments. These ideas are, briefly, described below**:**

Modeling of Child Growth and Obesity

Our group has been involved in modeling of child growth and obesity since 2002. Initial modeling assumed that height and mass trajectories were, approximately, linear if the measurements were performed 6-*month* apart, with the exception of regions characterized by rapid change in growth rate (infancy-to-childhood phase and childhood-to-puberty phase). Linear interpolation was used to compute height at some age grid to be compared with current-age-mid-parental height to determine whether the child was tall or stunted. Similar procedure was employed to figure out if the child was obese or wasted (Kamal *et al.*, 2002; 2004), This model had the drawback that two checkups were needed to generate a profile, not recommended for children requiring immediate intervention. A limited 'Growth-and-Obesity Profile', based on a single checkup, was generated employing the technique of 'box interpolation' (Kamal *et al.*, 2011). Last year a method to generate 'Growth-and-Obesity Roadmap' (called 'Scalar-Roadmap' in

Table 4. Lifestyle adjustment, diet and exercise plans for a child to achieve month-wise targets

	Height Management[#]	*Mass (Weight) Management*[#]
Lifestyle Adjustment	Recommended daily dose of vitamin D (600 IU) through 10-15 minute guarded-graduated[∈] sun-exposure (early morning or late afternoon) with the child minimally dressed (leaving head, arms, legs and spinal column exposed, last one from external auditory meatus to hip joint; eyes protected through UV-cutoff glasses); 8-hour, night-time, sound sleep dressed in pajama shorts only[@] (3-minute, slow-stoke back massage to improve quality and quantity of sleep); maximum 2-hour screen time (one hour computer/video games — computer monitor at eye level; one hour TV/DVD) — light exercises during TV/DVD watching; for 5 minutes each after waking up, at the end of every hour and before going to bed — bend-ing on sides, focusing eyes far away and moving eyeballs, moving fingers and wrists after computer work and writing, stretching, touching toes without flexing knees, exercising neck muscles (left, right, up, down)	
Diet Plans	3 relaxed and balanced meals, 10-12 glasses of water daily — absolutely NO carbonated drinks[&]	
	To gain height, diet plan should include calcium-, protein- and fiber-rich diet (milk, fresh fruit, chicken and fish)	To put on mass (weight), diet plan should include milk, potato items (baked or boiled, but not fried) and protein-rich diet; to shed off mass (weight), diet plan should include salad, yogurt and skimmed milk
Exercise Plans	Guarded-graduated[©] exercises preceded by warm-up and followed by cool-down routines, preferably outdoors (weather permitting) in exercise-friendly clothing[ɜ]	
	To pick up height, child should perform light-stretching exercises (bar hanging, mild-stretching, summersault, cartwheel)	To increase mass (weight), heavy exercises performed for shorter duration, consistently; to lose mass (weight), child should perform light exercises for longer duration, consistently

[#]Lifestyle-adjustment guidelines are taken from (Kamal *et al.*, 2013*a*), height-management from (Kamal *et al.*, 2013*b*) and mass-management from (Kamal *et al.*, 2013*e*).

[∈]'Guarded' implies surveillance of overexposure, which may produce skin burn (short term) and skin cancer (long term); 'graduated' means systematic increase in exposure for body conditioning (Kamal & Khan, 2015).

[@]Sleeping in day clothes or underwear should be discouraged. In gender-segregated sleeping quarters, boys of all ages and younger girls should be encouraged to sleep stripped-to-waist, allowing the body to breathe and increasing tactile stimulation (Kamal & Khan, 2014).

[&]Carbonated drinks take away body's capacity to absorb calcium and iron and hence should be avoided, not only, by children, but also, by persons of all ages, in particular, older individuals.

[ɜ]Details of exercise-friendly clothing are given in Kamal & Khan (2015).

[©]Guarded-graduated exercises should contribute towards health- as well as skill-related fitness (performance considerations). Such practices, also, avoid exercise-related injuries (safety considerations). 'Guarded' is related to the concept that different body ligaments are in stable equilibrium, locally, during different exercise phases and 'graduated' implies that sequential exercise phases are related by infinitesimal transformations (Kamal & Khan, 2013).

this work to differentiate from 'Vector-Roadmap') was put forward (Kamal, 2015*a*, *b*; Kamal *et al.*, 2015).

- There is a need to extend the vector model to still-growing parents on the pattern of Kamal and Jamil (2012).
- In the vector model, the projected percentile curve (dotted portion) is smooth (both continuous and differentiable) at the terminal point (selected to lie at the age of 10 *years*). However, at the start of intervention (age of the most-recent checkup), taken as the initial point, it is continuous but not smooth. It is desirable to smoothen the (height/mass) percentile curve at the initial point in addition to the terminal point, so that the percentile curve (actual + recommended) remains differentiable throughout the region. This may be accomplished by looking at the form of the radiative-transfer equation — the basis of CT scanner (Rybici and Lightman, 1979)

$$\frac{dI_\nu(\tau_\nu)}{d\tau_\nu} + I_\nu(\tau_\nu) = S_\nu(\tau_\nu)$$

where $I_\nu(\tau_\nu)$ is the intensity as a function of optical depth, τ_ν (ν being the frequency), $S_\nu(\tau_\nu)$ is the source

function. The reference trajectory is $I_v(\tau_v) = S_v(\tau_v)$. For $\tau_v >> 1$, $I_v(\tau_v) \to S_v(\tau_v)$.

Growth Charts

The major problem encountered by the NGDS Team at the time of start of anthropometric-data collection (1998) was the non-availability of growth charts of the Pakistani children. Hence, it was decided to use CDC Growth Charts and Tables with appropriate scaling and adjustments — children falling below 40th *percentile* of CDC Growth Charts, instead of 50th *percentile*, were considered stunted (Kamal *et al.*, 2015); current-age-mid-parental height taken as reference (Kamal *et al.*, 2002; 2004). Still many children had their height and masses either below 3rd percentile or above 97th percentile, whose profiles could not be determined using regular CDC percentiles. Hence, special methods to deal with extreme cases were developed (Kamal *et al.*, 2011; Kamal and Jamil, 2012). However, a major breakthrough was extending these charts on both sides to include 0.01th, 0.1th and 1st as well as 99th, 99.9th and 99.99th *percentiles* (Kamal and Jamil, 2014).

- Further extension of CDC Growth Charts is needed to include 0.001th and 99.999th percentiles following the strategies of Kamal and Jamil (2014).
- Growth charts are required for transgender children in response to increased awareness for rights of this segment of population.
- Although there have been some isolated efforts to generate growth charts for the Pakistani children (Aziz *et al.*, 2012; Mushtaq *et al.*, 2012), a more dedicated endeavor is called for in order to construct and regularly update such charts.

Enhancement of Anthropometric Instruments

Growth-and-Obesity Roadmaps shall not be reliable if height and mass measurements are not properly taken. The NGDS Team has worked on standardization of anthropometric techniques (Kamal *et al.*, 2013*g*) and prepared training manuals for in-house use and external trainings (Kamal, 2006*b*). In addition, efforts have been made to develop instruments in SF-Growth-and-Imaging Laboratory (wall-mounted-height-measurement system; moiré frame to be used in enhancement of height- and mass-measurement systems) as well as improve existing equipments (Kamal, 1982; 2010) — at present heights and masses are measured to least counts of 0.005 *cm* and 0.005 *kg*, respectively (Kamal, 2016*a*). Further enhancement of height- and mass-recording systems may be achieved along the following lines**:**

- Using circular scale (similar to the one employed in micrometer screw gauge) instead of Vernier scale to reduce height-scale least count from 0.005 *cm* to 0.001 *cm*. A similar mechanism could be utilized to decrease beam-scale least count from 0.005 *kg* to 0.001 *kg*. However, it should be noted that for enhancing mass-measurement system, there is a constraint to match of mass of circular scale with mass of the sliding weight present in regular beam scale.
- Wheatstone-bridge-type system may be used to ensure that the child exerts equal weight on both feet during height and mass measurement.
- Symmetric moiré (Kamal *et al.*, 2013*c*) of back (about the sagittal plane) during mass recording may be used to ascertain proper posture is maintained during the procedure — moiré fringe topography is a non-contact, non-invasive, 3-D-optical technique, which generates contours of constant surface elevation (normal to frontal plane) of human back.
- Symmetric moiré of front (about the sagittal plane) during height measurement may be utilized to make certain upright posture is achieved during this phase of anthropometry.

CONCLUSION

In this work childhood-obesity problem has been scrutinized from perspectives of physics and mathematics. The scalar model has been replaced by the vector model — the resulting roadmap termed as 'Growth-and-Obesity Vector-Roadmap'. This map is generated from a series of height and mass measurements (least counts upgraded to 0.005 *cm* and 0.005 *kg* starting this year) obtained by reproducible anthropometrists using 'Extended CDC Growth Charts and Tables', which have heights and masses listed for extreme percentiles. This Vector-Roadmap is different from the Scalar-Roadmap (put forward in early 2015) in the sense that the child is required to pick up height or gain/lose mass gradually within the next 6 *months*, till the youngster enters peripubertal phase, age of 10 *years* in the context of this model, to avoid unnecessary stress on a child's skeleton realizing that height gain is a quasi-static process representing tissue synthesis. The softer targets of mass management during the time span of the next half year, generated by the vector model, should, hopefully, be easier to achieve for the children assisted by their caregivers.

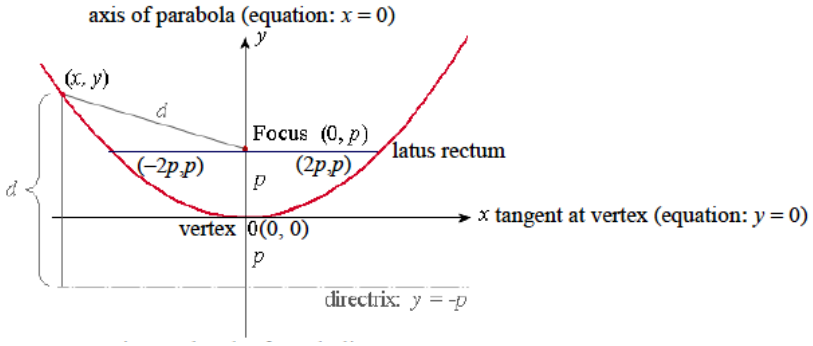

Fig. 4. Sketch of the parabolic curve

ACNOWLEDGEMENTS

The authors would like to express their deepest appreciation for Prof. Dr. Muhammed Qaiser, Vice Chancellor, University of Karachi, for his patronage and Prof. Dr. Syed Afrozuddin Ahmed, Professor of Statistics and Dean, Faculty of Science, University of Karachi for taking interest in the group's work. AAN and SAA are indebted to Prof. Dr. Shahid Mahmood, Professor and Chairman, Department of Physics, University of Karachi for facilitating their research endeavors. Special thanks are due to Dr. Yasmin Ansar Rizvi, Medical Officer, University-of-Karachi Clinic, for convincing the first author to recommend banning carbonated drinks in children. Some of the graphs included in this paper were drawn by Dr. Maqsood Sarwar, PhD, Anthromathematics, University of Karachi. Saba Musafar, Visiting Faculty, Department of Mathematics, University of Karachi constructed enhanced anthropometric instruments used during the fourth checkup of L. G.

APPENDIX A: EQUATION OF PARABOLA

The parabola is defined as the curve generated from the locus of a point, which moves in such a manner so that it remains, always, at the same distance from a fixed point (termed as 'focus') and a given line (termed as 'directrix'). 'Latus rectum' is the line segment parallel to directrix passing through focus with end points on the parabolic curve. 'Axis of symmetry' (also called 'axis of parabola') is the line normal to directrix passing through focus (Figure 4). Derivations of equations of parabola are available in standard mathematics books. Since the paper is, also, of interest to audiences of pure biological science (non-mathematical) background, we have included derivation of equation of parabola on page 11 of Additional File 3.

The first and the third formats of equations, given in Tables 4*a*, *b*, are used in fitting height and mass percentile equations. Let us now transform these equations to represent parabolic curves, where the age, *A* (*years*), takes up the role of *x* coördinate, and percentile, *P*, assumes the role of *y* coördinate. The vertex of parabola lies at $(A_{\text{ref}}, P_{\text{ref}}) = (10, P_{\text{ref}})$. The equation of parabola opening upward

(A1) $$(x - x_0)^2 = 4p(y - y_0)$$

becomes

(A2) $$(A - 10)^2 = 4p(P - P_{\text{ref}})$$

Table 4*a*. Properties of parabola with vertex at origin (0, 0)

Equation of parabola	*Equation of axis of symmetry*	*Tangent to parabola at vertex (0, 0)*	*Equation of directrix*	*Coördinates of focus*	*End-points of latus rectum (length = 4p)*	*Opening*
$x^2 = 4py$[⊂]	$x = 0$[⊃]	$y = 0$	$y = -p$	$(0, p)$	$(-2p, p), (2p, p)$	upward
$y^2 = 4px$	$y = 0$	$x = 0$	$x = -p$	$(p, 0)$	$(p, -2p), (p, 2p)$	right
$x^2 = -4py$	$x = 0$	$y = 0$	$y = p$	$(0, -p)$	$(-2p, -p), (2p, -p)$	downward
$y^2 = -4px$	$y = 0$	$x = 0$	$x = p$	$(-p, 0)$	$(-p, -2p), (-p, 2p)$	left

[⊂]This parabola is drawn in Figure 4.
[⊃]The axis of symmetry is *y*-axis, whose equation is shown in the second column.

Table 4*b*. Parabola properties with vertex at (x_0, y_0)

Equation of parabola	*Equation of axis of symmetry*	*Tangent to parabola at vertex* (x_0, y_0)	*Equation of directrix*	*Coördinates of focus*	*End-points of latus rectum* (*length* = 4*p*)	*Opening*
$(x-x_0)^2 = 4p(y-y_0)$	$x = x_0$	$y = y_0$	$y = y_0 - p$	$(x_0, y_0 + p)$	$\begin{cases}(x_0 - 2p, y_0 + p),\\(x_0 + 2p, y_0 + p)\end{cases}$	upward
$(y-y_0)^2 = 4p(x-x_0)$	$y = y_0$	$x = x_0$	$x = x_0 - p$	$(x_0 + p, y_0)$	$\begin{cases}(x_0 + p, y_0 - 2p),\\(x_0 + p, y_0 + 2p)\end{cases}$	right
$(x-x_0)^2 = -4p(y-y_0)$	$x = x_0$	$y = y_0$	$y = y_0 + p$	$(x_0, y_0 - p)$	$\begin{cases}(x_0 - 2p, y_0 - p),\\(x_0 + 2p, y_0 - p)\end{cases}$	downward
$(y-y_0)^2 = -4p(x-x_0)$	$y = y_0$	$x = x_0$	$x = x_0 + p$	$(x_0 - p, y_0)$	$\begin{cases}(x_0 - p, y_0 - 2p),\\(x_0 - p, y_0 + 2p)\end{cases}$	left

The above equation may be written as

(A3) $$(P - P_{\text{ref}}) = \frac{1}{4p}(A-10)^2$$

Such an equation can only be fitted to percentile of mass, when its value exceeds the value of reference percentile. To compute the value of *p*, note that the parabolic curve starts at age of the most-recent checkup (age of intervention), A_0, when the percentile of mass is $P(\mu, A_0)$, a point on the parabola is found as $(A_0, P(\mu, A_0))$. Substituting these coördinates in equation (A3), the value of $1/(4p)$ is obtained as

(A4) $$\frac{1}{4p} = \frac{P(\mu, A_0) - P_{\text{ref}}}{(A_0 - 10)^2}$$

Substituting this value in (A2) gives equation (2*b*). A similar derivation for parabola opening downward is applicable to percentiles of height and mass, when their values are lesser as compared to reference percentile at age of the most-recent checkup — equations (1*b*) and (2*b*). When any of these percentiles matches the reference percentile, equations (1*a*) or (2*a*) are applicable. Table 4*c* summarizes the parameters of parabolic equations written in terms of percentiles, $P(h, A_0)$ and $P(\mu, A_0)$, as functions of age, *A*.

APPENDIX B: ADDITIONAL RESOURCES

Additional File 1 (http://www.ngds-ku.org/Papers/J43/Additional_File_1.pdf) contains description of institutional review process, description of the NGDS checkups on school premises ('Informed Consent Form', instruction slip, rationale for the instructions, preparations and 'Master data Sheet' used to record findings), description of the SGPP checkups ('The SGPP Participation Form', appointment letter, rationale for instructions, history form and examination form) as well as virtual tour of the SF-Growth-and-Imaging Laboratory (specialized facilities, equipment and software available).

Additional File 2 (http://www.ngds-ku.org/Papers/J43/Additional_File_2.pdf) contains report given to parents of L. G. based on the scalar model. On page 5 confidentiality standards are described. In addition, dress and behavior codes are explained.

Additional File 3 (http://www.ngds-ku.org/Papers/J43/Additional_File_3.pdf) contains report given to parents of L. G. based on the vector model. At the end of report, summary of history, physical examination and clinical photographs of L. G. are included. Pages 7-10 give color codes used in Growth-and-Obesity Vector-Roadmaps shown on pages 2-4. Equation of parabola is derived on page 11 and desired trajectories computed on page 12.

APPENDIX C: COMPLIANCE WITH ETHICAL STANDARDS

Conflict of Interest

The authors declare that they don't have any financial/non-financial competing interests in the research presented in this work.

Table 4c. Parabolic equations fitted to percentile curves

Description	*Equation of parabola*	*Equation of axis of symmetry*	*Tangent to parabola at vertex* $(10, P_{ref})$	*Equation of directrix*	*Coördinates of focus*	*End-points of latus rectum*
Mass-percentile curve opening upward	$P(\mu,A) = P_{ref} - (P_{ref} - P(\mu,A_0))\left(\frac{A-10}{A_0-10}\right)^2$	$A = 10$	$P(\mu,A) = P_{ref}$	$P(\mu,A) = P_{ref} - \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}$	$\left(10, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)$	$\left\{\left(A_0 - \frac{(A_0-10)^2}{2(P(\mu,A_0)-P_{ref})}, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)\right.$ $\left.\left(A_0 + \frac{(A_0-10)^2}{2(P(\mu,A_0)-P_{ref})}, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)\right\}$
Mass-percentile curve opening downward	$P(\mu,A) = P_{ref} - (P_{ref} - P(\mu,A_0))\left(\frac{A-10}{A_0-10}\right)^2$	$A = 10$	$P(\mu,A) = P_{ref}$	$P(\mu,A) = P_{ref} - \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}$	$\left(10, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)$	$\left\{\left(A_0 - \frac{(A_0-10)^2}{2(P(\mu,A_0)-P_{ref})}, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)\right.$ $\left.\left(A_0 + \frac{(A_0-10)^2}{2(P(\mu,A_0)-P_{ref})}, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)\right\}$
Height-percentile curve opening downward	$P(h,A) = P_{ref} - (P_{ref} - P(h,A_0))\left(\frac{A-10}{A_0-10}\right)^2$	$A = 10$	$P(h,A) = P_{ref}$	$P(h,A) = P_{ref} - \frac{(A_0-10)^2}{4(P(h,A_0)-P_{ref})}$	$\left(10, P_{ref} + \frac{(A_0-10)^2}{4(P(h,A_0)-P_{ref})}\right)$	$\left\{\left(A_0 - \frac{(A_0-10)^2}{2(P(\mu,A_0)-P_{ref})}, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)\right.$ $\left.\left(A_0 + \frac{(A_0-10)^2}{2(P(\mu,A_0)-P_{ref})}, P_{ref} + \frac{(A_0-10)^2}{4(P(\mu,A_0)-P_{ref})}\right)\right\}$

Institutional Review

The NGDS Pilot Project was initiated in 1998 under of directives of Governor Sindh/Chancellor, University of Karachi after proper institutional review process. It was designed after considering North American and European, ethical and human-right standards. Additional File 1 gives the details.

Informed Consent

'The Informed Consent Form' for school studies, based on opt-in policy is included in Additional File 1 and is, also, available at**:**

http://www.ngds-ku.org/ngds_folder/Protocols/NGDS_form.pdf

'The SGPP Participation Form' for detailed checkups in the SF-Growth-and-Imaging Laboratory is, also, part of Additional File 1. In addition, it is uploaded at**:** http://www.ngds-ku.org/SGPP/SGPP_form.pdf. Both forms required signatures of both parents as well as their child(ren). Before the start of checkup, verbal permission was sought from the child(ren) and the attending parent(s).

Privacy, Confidentiality, Comfort and Safety

Both visual as well as acoustic privacy is offered in the SF-Growth-and-Imaging Laboratory. Doors are closed and locked and second level of privacy is offered through a curtained-off area.

Family labels and children's initials presented in this manuscript and the supplementary documents do not correspond to first letters in actual names (as per confidentiality standards established by the NGDS Team). Same is true about case numbers appearing in the main and the additional material. These are different from the numbers entered in reports given to parents. Further, in place of scanned signatures, initials are given, again, to protect privacy. Additional File 2 gives the details.

Comfort of patients is of prime concern. Fans are not turned on even in mild weather, as children are undressed for checkups. In colder weather, examiner warms hands before palpation and percussion. In addition, bell and diaphragm of stethoscope are warmed before auscultation. Although, both parents are invited to the checkups to share history and progress (heights of biological mother and biological father are recorded on first visit to compute target height), same-gender parent is preferred to be present at the actual checkup in the curtained-off area for maximum comfort of the child.

Prior to checkups, school-checkup-room floor was cleaned to prevent any sharp objects on floor, which could hurt barefooted students, while their gait was analyzed. Benches/chairs were checked for sharp wood edges as well as both sides of the mounted engineering tape to prevent skin abrasions and cuts to undressed pupils. In SF-Growth-and-Imaging Laboratory, the entire floor is black-tiled. Outside shoes are not allowed for the children, their parents and the laboratory staff. Floor is tided and mopped with dettol-mixed water (generic name of dettol is chloroxylenol). Bulbs of thermometers, when not in use, are kept dipped in dettol-mixed water. Hand washing and sanitization is mandatory at the beginning of checkup. Health professionals and anthropometrists are required to remove hand-worn chains, rings and wristwatches to prevent injury to examinees.

REFERENCES

Aslam, S., Y. Iqbal and A. Ghouri (2013). Community beliefs, attitude and awareness about child obesity. *The Shield (International Journal of Physical Education and Sport Science)*, 8**:** 1-25

Aziz, S., W. Noor-ul-Ain, R. Majeed, M. A. Khan, I. Qayam, I. Ahmed and K. Hosain (2012). Growth centile charts (anthropometric measurement) of Pakistani pediatric population. *Journal of Pakistan Medical Association*, 62**:** 367-377

Baker, J. L., L. W. Olsen and T. I. A. Sørensen (2007). Childhood body-mass index and risk of coronary heart disease in adulthood. *New England Journal of Medicine*, 357**:** 2329-2337

Bibbins-Domingo, K., P. Coxson, M. J. Pletcher, J. Lightwood and L. Goldman (2007). Adolescent overweight and future coronary heart disease. *New England Journal of Medicine*, 357**:** 2371-2379

Covic, T., L. Roufiel and S. Dziurawiec (2007). Community beliefs about childhood obesity**:** Its causes, consequences and potential solutions. *Journal of Public Health*, 29**:** 123-131

Finegood, D. T., T. D. N. Merth and H. Rutter (2010). Implications of the Foresight Obesity System Map for solutions to childhood obesity. *Obesity*, 18 (Supplement)**:** S13-S16

Han, J. C., D. A. Lawlor and S. Y. S. Kimm (2010). Childhood obesity. *Lancet*, 375**:** 1737-1748

Hunsberger, M., S. Lehtinen-Jacks, K. Mehlig *et al.* (2016). Bidirectional associations between psychological well-being and body mass index in European children: Longitudinal findings from the IDEFICS study. *BMC Public Health*, 16**:** doi**:** 10.1186/s12889-016-3626-4

Kamal, S. A. (1982, April 26-29). Improvements in instruments for anthropometric measurements. *Spring Meeting of the American Physical Society*, Washington, DC, United States; *Bulletin of the American Physical Society*, 24**:** 502 (poster presentation), abstract#DYc4**:** http://www.ngds-ku.org/pub/confabst1.htm#C19:

Kamal, S. A. (2002, June 10-15). Incompleteness of cross-product steering and a mathematical formulation of extended-cross-product steering. *Proceedings of the First International Bhurban Conference on Applied Sciences and Technologies (IBCAST 2002)*, Volume 1, Advanced Materials, Computational Fluid Dynamics and Control Engineering, edited by H. R. Hoorani, A. Munir, R. Samar and S. Zahir, National Center for Physics, Islamabad, Pakistan, pp. 167-177, full text**:** http://www.ngds-ku.org/Papers/C56.pdf

Kamal, S. A. (2006*a*, April 21, 22). Cross-range error in the Lambert scheme. *Proceedings of the Tenth National Aeronautical Conference*, edited by S. R. Shaikh and A. M. Khan, PAF Academy, Risalpur, KP, Pakistan, pp. 255-263, full text**:** http://www.ngds-ku.org/Papers/C67.pdf

Kamal, S. A. (2006*b*, May 23). *Manual for Obtaining Anthropometric Measurements*. University of Karachi, Karachi, Pakistan, version 9.1 (release date**:** April 5, 2016), full text**:** http://www.ngds-ku.org/ngds_folder/M02.pdf

Kamal, S. A. (2010, March 22). Mathematics in the life sciences. *The First National Conference on Mathematical Sciences*, Golden Jubilee Celebration, Department of Mathematics, University of Karachi, Karachi, Pakistan, p. 19 (the Syed Firdous memorial lecture), abstract#10**:** http://www.ngds-ku.org/Presentations/Firdous.pdf

Kamal, S. A. (2014, March 20). The extended-Q system. *The Second Conference on Mathematical Sciences (CMS 2014)*, Department of Mathematics, University of Karachi, Karachi, Pakistan, p. 3 (lecture in honor of Richard H. Battin), abstract#CMS14-12**:** http://www.ngds-ku.org/Presentations/ExtendedQ.pdf

Kamal, S. A. (2015*a*). Optimal-mass management in obese children. *International Journal of Biology and Biotechnology*, 12**:** 381-391, full text**:** http://www.ngds-ku.org/Papers/J38.pdf

Kamal, S. A. (2015*b*). Acute malnutrition in a child suffering from cardiac problems. *International Journal of Biology and Biotechnology*, 12**:** 585-600, full text**:** http://www.ngds-ku.org/Papers/J40.pdf

Kamal, S. A. (2015*c*, December 28-30). US-childhood-obesity problem**:** Solutions proposed in Pakistan. *The First International Conference on Life Sciences (Emerging Trends in Biological Sciences and Genomics)*, MAH Qadri Biological Research Center, University of Karachi, Karachi, Pakistan (invited lecture), pp. 8, 9 abstract**:** http://www.ngds-ku.org/Presentations/Childhood-Obesity.pdf

Kamal, S. A., A. A. Naz, S. Musafar and S. A. Ansari (2016*a*, February 12, 13). Growth-and-Obesity Vector-Roadmaps using enhanced anthropometric instruments**:** The fourth-generation solution of childhood obesity. *The Karachi Physics Society First National Conference on Multidisciplinary Topics in Physics*, Department of Physics, University of Karachi, Karachi, Pakistan, p. 56 (invited lecture), abstract#IT-04**:** http://www.ngds-ku.org/Presentations/Vector.pdf

Kamal, S. A. and A. Mirza (2006). The multi-stage-Q system and the inverse-Q system for possible application in satellite-launch vehicle. *Proceedings of the Fourth International Bhurban Conference on Applied Sciences and Technologies, June 13-18, 2005 (IBCAST 2005)*, Volume 3, Control and Simulation, edited by S. I. Hussain, A. Munir, J. Kayani, R. Samar and M. A. Khan, National Center for Physics, Islamabad, Pakistan, pp. 27-33, full text**:** http://www.ngds-ku.org/Papers/C66.pdf

Kamal, S. A., C. M. S. Manzoor and S. A. Khan (2013*a*, September 4, 5). Diet-based interventions and vitamin-D deficiency. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 14, abstract#Anthro13-08**:** http://www.ngds-ku.org/Presentations/VitaminD.pdf

Kamal, S. A., C. M. S. Manzoor and S. A. Khan (2013*b*, September 4, 5). Increasing height through diet, exercise and lifestyle adjustment, *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 18, abstract#Anthro13-12**:** http://www.ngds-ku.org/Presentations/Height_Management.pdf

Kamal, S. A., M. K. Rajput and S. A. Ansari (2016*b*). Gait analysis of 7-10-year-old children of Karachi from nutritional-status perspective. *International Journal of Biology and Biotechnology*, 13**:** 13-25, full text**:**

http://www.ngds-ku.org/Papers/J41.pdf

Kamal, S. A., M. Sarwar and S. K. Raza (2013*c*, September 4, 5). Moiré fringe topography in scoliosis detection and management. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 16, abstract#Anthro13-10**:** http://www.ngds-ku.org/Presentations/Moire.pdf

Kamal, S. A., M. Sarwar and U. A. Razzaq (2013*d*, September 4, 5). Cumulative-Scoliosis-Risk Weightage (CSRW) — designing preventive strategies. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 11, abstract#Anthro13-05**:** http://www.ngds-ku.org/Presentations/CSRW.pdf

Kamal, S. A., M. Sarwar and U. A. Razzaq (2015*a*). Effective decision making for presence of scoliosis. *International Journal of Biology and Biotechnology*, 12**:** 317-328, full text**:** http://www.ngds-ku.org/Papers/J36.pdf

Kamal, S. A., N. Jamil and S. A. Khan (2011). Growth-and-Obesity Profiles of children of Karachi using box-interpolation method. *International Journal of Biology and Biotechnology*, 8**:** 87-96, full text**:** http://www.ngds-ku.org/Papers/J29.pdf

Kamal, S. A., S. Burki and S. S. Jamil (2013*e*, September 4, 5). Optimal-weight management through diet, exercise and lifestyle adjustment. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 9, abstract#Anthro13-03**:** http://www.ngds-ku.org/Presentations/Optimal_Weight.pdf

Kamal, S. A., S. Firdous and S. J. Alam (2004). An investigation of growth profiles of the Pakistani children. *International Journal of Biology and Biotechnology*, 1**:** 707-717, full text**:** http://www.ngds-ku.org/Papers/J26.pdf

Kamal, S. A., S. J. Alam and S. Firdous (2002, June 22). The NGDS Pilot Project**:** Software to analyze growth of a child (a telemedicine perspective). *National Telemedicine Conference Pakistan 2002*, Technology Resource Mobilization Unit (TreMU), Ministry of Science and Technology, Government of Pakistan, Islamabad, Pakistan, p. 2, full text**:** http://www.ngds-ku.org/Papers/C52.pdf

Kamal, S. A., S. A. Ansari and M. Sarwar (2016*c*). Medical criteria for induction into the Armed Forces of Pakistan**:** Cutoff heights for still-growing youth. *Pakistan Armed Forces Medical Journal* (accepted for publication

Kamal, S. A., S. A. Ansari and S. S. Jamil (2014*a*, September 4). Growth-and-Obesity Enhanced-Roadmaps of children. *The Second Conference on Anthromathematics and Sport Mathematics in the Memory of (Late) Hussain Ahmed Bilgirami (ANTHROMATHEMATICS 2014)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 10 (concluding talk), abstract#Anthro14-06**:** http://www.ngds-ku.org/Presentations/Enhanced.pdf

Kamal, S. A., S. A. Ansari and S. S. Jamil (2015*b*). Generating and validating Growth-and-Obesity Roadmaps for the Pakistani children. *International Journal of Biology and Biotechnology*, 12**:** 47-61, full text**:** http://www.ngds-ku.org/Papers/J35.pdf

Kamal, S. A. and S. A. Khan (2013, September 4, 5). Fitness for primary-school children. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 24, abstract# Anthro13-18**:** http://www.ngds-ku.org/Presentations/Fitness.pdf

Kamal, S. A. and S. A. Khan (2014). Primary-physical-education practices in Pakistan and England**:** Health and safety perspectives. *International Journal of Biology and Biotechnology*, 11**:** 401-419, full text**:** http://www.ngds-ku.org/Papers/J33.pdf

Kamal, S. A. and S. A. Khan (2015). Hairstyle, footwear and clothing for gymnastic activities in the primary-school setting. *Pumukkale Journal of Sport Sciences*, 6**:** 46-64, full text**:** http://www.ngds-ku.org/Papers/J37.pdf

Kamal, S. A., S. Firdous and S. J. Alam (2004). An investigation of growth profiles of the Pakistani children. *International Journal of Biology and Biotechnology*, 1**:** 707-717, full text**:** http://www.ngds-ku.org/Papers/J26.pdf

Kamal, S. A., S. K. Raza and M. Sarwar (2016*d*). Mathematical modelling of scoliosis indicators in growing children. *International Journal of Biology and Biotechnology*, 13**:** 471-484, full text**:** http://www.ngds-ku.org/Papers/J42.pdf

Kamal, S. A. and S. S. Jamil (2012). A method to generate Growth-and-Obesity Profiles of children of still-growing parents. *International Journal of Biology and Biotechnology*, 9**:** 233-255, full text**:** http://www.ngds-ku.org/Papers/J30.pdf

Kamal, S. A. and S. S. Jamil (2014). KJ-regression model to evaluate optimal masses of extreme cases. *International Journal of Biology and Biotechnology*, 11**:** 623-648, full text**:** http://www.ngds-ku.org/Papers/J34.pdf

Kamal, S. A., S. S. Jamil and S. A. Ansari (2013*f*, September 4, 5). Growth-and-Obesity Roadmaps of children. *The First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, p. 8, abstract#Anthro13-02**:** http://www.ngds-ku.org/Presentations/Roadmap.pdf

Kamal, S. A., S. S. Jamil and U. A. Razzaq (2014*b*). Stunting induced by wasting — Wasting induced by stunting**:** A case study *International Journal of Biology and Biotechnology*, 11**:** 147-153, full text**:** http://www.ngds-ku.org/Papers/J32.pdf

Kamal, S. A., U. A. Razzaq and S. S. Jamil (2013*g*, September 4, 5). Importance of standardization of anthropometric protocols. Presentation at *the First Conference on Anthromathematics in the Memory of (Late) Syed Firdous (ANTHROMATHEMATICS 2013)*, Department of Mathematics, University of Karachi, Karachi, Pakistan and Government College, Hyderabad, Pakistan, 12, abstract#Anthro13-06**:** http://www.ngds-ku.org/Presentations/Anthropometry.pdf

Karlberg, J. (1987). Modeling of human growth. *PhD Dissertation*, University of Göteborg, Göteborg, Sweden

Keys, A., F. Fidanza, M. J. Karvonen, N. Kimura, H. L. Taylor (1972). Indices of relative weight and adiposity. *Journal of Chronic Diseases*, **25:** 329-343

Kyle, T. K., E. J. Dhurandhar and D. B. Allison (2016). Regarding obesity as a disease**:** Evolving policies and implications. *Endocrinology and Metabolism Clinics of North America*, 45**:** 511-520

Lloyd, L. J., S. C. Langley-Evans and S. McMullen (2010). Childhood obesity and adult cardiovascular disease risk**:** A systematic review. *International Journal of Obesity*, 34**:** 18-28

Ludwig, D. S. (2007). Childhood obesity — the shape of things to come. *New England Journal of Medicine*, 357**:** 2325-2327

Miller, J. L. and J. H. Silverstein (2007). Management approaches for pediatric obesity. *Nature Clinical Practice in Endocrinology and Metabolism*, 3**:** 810-818

Miqueleiz, E., L. Lostao and E. Regidor (2016). Stabilization of the trend in prevalence of childhood overweight and obesity in Spain**:** 2001-11. *European Journal of Public Health*, doi**:** 10.1093/eurpub/ckw087

Miqueleiz, E., L. Lostao, P. Ortega, J. M. Santos, P. Astasio and E. Regidor (2014). Trends in prevalence of childhood overweight and obesity according to socioeconomic status**:** Spain, 1987-2007. *European Journal of Clinical Nutrition*, 68**:** 209-214

Mushtaq, M. U., S. Gull, K. Mushtaq *et al.* (2012). Height, weight, *BMI* percentiles and nutritional status relative to the international growth reference among Pakistani school-aged children. *BMC Pediatrics*, 12**:** 31-42

Ng M., T. Fleming, M. Robinson *et al.* (2014). Global, regional, and national prevalence of overweight and obesity in children and adults during 1980–2013**:** A systematic analysis for the Global Burden of Disease Study 2013. *The Lancet*, 384**:** 766-781

Nierop, A. F. M., A. Niklasson, A. Holmgren, L. Gelander, S. Rosberg and K. Albertsson-Wikland (2016). Modeling individual longitudinal human growth from fetal to adult life — QEPS I. *Journal of Theoretical Biology*, 406**:** 143-165

Olds, T. S., G. R. Tomkinson, K. E. Ferrar and G. A. Maher (2010). Trends in prevalence of childhood overweight and obesity in Australia between 1985 and 2008. *International Journal of Obesity*, 34**:** 57-66

Ogden, C. L., M. D. Carroll, B. K. Kit and K. M. Flegal (2014). Prevalence of childhood and adult obesity in the United States, 2010-2012. *Journal of the American Medical Association*, 311**:** 806-814

Pedersen, D. C., J. Aarestrup, S. Pearson and J. L. Baker (2016). Ethnic inequalities in overweight and obesity prevalence among Copenhagen schoolchildren from 2002 to 2007. *Obesity Facts*, 9**:** 284-295

Ramzan, M., I. Ali, A. S. Khan (2008). Body-mass status of school children of Dera Ismail Khan, Pakistan. *Journal of Ayub Medical College*, 20**:** 119-121

Robinson, T. N. and J. R. Sirard (2005). Preventing childhood obesity**:** A solution-oriented research paradigm. *American Journal of Preventive Medicine*, 28**:** 194-201

Rybicki, G. B. and A. P. Lightman (1979). *Radiative Processes in Astrophysics*. John Wiley**:** New York, United States, 17

Shafqat, N., S. Manzoor and S. Abbasi (2013). Relationship of sociodemographic factors with malnutrition in preschool children**:** A community based study. *Pakistan Journal of Medical Research*, 52**:** 120-125

Silventoinen, K., B. Rockholm, J. Kaprio and T. I. A. Sørensen (2010). The genetic and environmental influences on childhood obesity**:** a systematic review of twin and adoption studies. *International Journal of Obesity*, 34**:** 29-40

Sørensen, T. I. A. (2009). Challenges in the study of causation of obesity. *Proceedings of the Nutrition Society*, 68**:** 43-54

Stamatakis, E., J. Wardle and T. J. Cole (2010). Childhood obesity and overweight prevalence trends in England**:** evidence for growing socioeconomic disparities. *International Journal of Obesity*, 34**:** 41-47

Tanner, J. M., H. Goldstein and R. H. Whitehouse (1970). Standards for children's height at ages 2-9 years allowing for height of parents. *Archives of Diseases of Children*, 45**:** 755-762

Whitaker, R. C., J. A. Wright, M. S. Pepe, K. D. Seidel and W. H. Dietz (1997). Predicting obesity in young adulthood from childhood and prenatal obesity. *New England Journal of Medicine*, 337**:** 869-873

Yanovski, S. Z. and J. A. Yanovski (2011). Obesity prevalence in the United States — up, down, or sideways? *New England Journal of Medicine*, 364**:** 987-989 (perspective)

Zhao, J. and S. F. A. Grant (2011). Genetics of childhood obesity. *Journal of Obesity*, article ID**:** 845148, doi**:** 10.1155/845148

*(Accepted for Publication**:** September 2016)*